An Impossibility Theorem in matching problems

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Summary. This paper studies the possibility of strategy-proof rules yielding satisfactory solutions to the matching problems. Alcalde and Barberá (1994) show that efficient and individually rational matching rules are manipulable in the one-to-one matching model. Thus, we pursue the possibility of strategy-proof matching rules by relaxing efficiency to a weaker condition of *respect for unanimity*. Our first result is positive. We prove that *there exists a strategy-proof rule which is individually rational and respects unanimity*. However, this rule often yields an unreasonable matching in the sense that a pair of agents who are the best for each other is not matched. Thus, in order to explore the possibility of better matching rules, we introduce a natural condition of "respect for pairwise unanimity." *Respect for pairwise unanimity* says that a pair of agents who are the best for each other should be matched, and an agent wishing to stay single should stay single. Our second result is negative. We prove that *there exists no strategy-proof rule which respects pairwise unanimity*. This result implies Roth (1982) showing that stable rules are manipulable. Furthermore, we extend our second result to the many-to-one matching model.