Uniform, equal division, and other envy-free rules between the two

By Takuma Wakayama

Faculty of Urban Liberal Arts, Tokyo Metropolitan University
E-mail: t_waka@center.tmu.ac.jp
URL: http://www.geocities.jp/takuma_wakayama/

Joint with
Toyotaka Sakai

Faculty of Economics and IGSSS, Yokohama National University

Abstract: This paper studies the problem of fairly allocating an amount of a divisible resource among agents whose preferences are single-peaked. Our purpose is to study various *envy-free* and *peak-only* rules and the structure of the set of those rules.

Since the class of *envy-free* and *peak-only* rules is quite large, we try to somehow compare the desirability of those rules. In Theorem 1, we show that, for every two *envy-free* and *peak-only* rules and every preference profile, the allocation chosen by one rule weakly Pareto dominates the allocation chosen by the other, and all agents are indifferent between the two allocations if and only if they are the same. Thus, given a preference profile, the Pareto dominance relation on the set of allocations chosen by *envy-free* and *peak-only* rules is a linear ordering. Furthermore, the unique greatest, least elements of the ordering are the uniform, equal division allocations, respectively.

We next consider ranking over rules. We say that a rule *dominates* another one if the allocation chosen by the former rule weakly Pareto dominates the allocation chosen by the latter at each and every preference profile. By definition, this dominance relation is a partial ordering over the set of *envy-free* and *peak-only* rules. In Theorem 2, we show that the set of *envy-free* and *peak-only* rules is a complete lattice with respect to this dominance relation, whose greatest, least elements are the uniform rule, and the equal division rule, respectively. Thus, as the title of this paper says, all *envy-free* and *peak-only* rules are "between" the uniform rule and the equal division rule. An immediate implication obtained from Theorem 2 is that, in the choice of *envy-free* and *peak-only* rules, there is no conflict among individual interests, and the uniform rule is unanimously considered to be best, while the equal division rule is worst.

We also show that, for every *envy-free* and *peak-only* rule and every preference profile, if the chosen allocation is neither the uniform allocation nor the equal division, its variance is more than the variance of the equal division and is less than the variance of the uniform allocation. This result greatly contrasts with Schummer and Thomson's (1997, *Economics Letters* **55**, 333–337, Proposition 2) result whereby the variance of the uniform allocation is always less than the variance of any other *efficient* allocation.