A new concept of mean-dependent intermediate inequality-equivalence

Tateo Yoshida[*]

This version: March 2007

Abstract. This paper introduces and characterizes a new concept of intermediate inequality, called *mean-dependent η-inequality equivalence.* Our new concept is a variant of *non-linear* intermediate inequality concept referred to as *η-inequality equivalence* in Yoshida (Soc Choice Welfare 24: 557-574; 2005). Although the weight $\eta \in [0,1]$ — which determines the degree of intermediateness between the "rightist" and the "leftist" inequality positions — is assumed constant in Yoshida(2005), it might quite reasonably be argued that people's attitudes towards inequality may be affected when the general level of incomes in the society changes. In this paper, we assume that the weight η is monotonically increasing and converges to 1 as the mean income of the society rises. Our notion reflects a view that, as a society becomes more affluent, inequality may become to be more "intensive" rather than to be more "equal-invariant".

The paper is organized as follows. Section 2 gives the formal definition of the mean-dependent η-inequality equivalence, and provides in Theorem 1 the necessary and sufficient condition for two income distributions to be inequality equivalent in the sense of our mean-dependent η-inequality equivalence. Section 3 is composed of two subsections: in Subsection 3.1, we identify in Theorem 2 the general form of the class of inequality measures satisfying the property of mean-dependent η-inequality equivalence; by use of this result, we show in Theorem 3 of Subsection 3.2 the equivalence between the unambiguous ordering generated by this class of inequality measures and the corresponding mean-dependent η-Lorenz dominance. In Section 4, we turn

our attention to the problem of installing our new concept of intermediate inequality in the form of equivalence between social welfare dominance and the mean-dependent η-Lorenz dominance. We assume a class of social evaluation functions satisfying Schur-concavity as well as the property that an increase in all incomes while keeping inequality intact in the sense of mean-dependent η-inequality criterion raises welfare. Then we show in Theorem 4 that an unambiguous ranking of two income distributions according to this class of social evaluation functions can be made if and only if one income distribution with a higher mean *dominates* the other in the sense of our mean-dependent η-Lorenz criterion.

$$
\begin{aligned}
&\vdots \qquad\qquad\qquad (\qquad\qquad\qquad\qquad (1)\\
&\vdots \\
&(\text{A new concept of mean-dependent intermediate inequality-equivalence})\\
&\vdots \qquad\quad (
\end{aligned}
$$