

# Comparative Statics of Weak-link Public Good Model\*

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## Abstract

This paper conducts several comparative static analyses of voluntary provision of a weak-link public good, the supply of which is determined by a concave social composition function. The social composition function determines how the level of a public good is determined by the contributions, and plays an important role especially in the context of international (or global) public goods ( see for example, Sandler (1997, 1998, 2002, 2003), Sandler and Arce (2002), and Ihori (1996)). However, many of the theoretical analyses of voluntary provision of public goods focuses on linear social composition functions. Relatively little attention has been paid on nonlinear social composition functions. Cornes and Hartley (2007b) investigate the voluntary provision of a public good with a CES social composition function. They show the existence and uniqueness of the Nash equilibrium and several properties of the replacement and share functions. As far as the author is aware, comparative static analysis in this case is quite limited.

One of the main contributions of the present paper lies in a decomposition of the responses of the best response function in terms of the price elasticity, marginal propensity of the best response function, and the substitution elasticity of the social composition function. Utilizing this decomposition enables us to conduct several comparative statics without specifying the functional form of the utility function. We investigate the effects of a multilateral income redistribution, a lump-sum transfer, a matching grant, and a price change. The results are summarized as following: (1) an income redistribution is neutral if one of the following conditions is satisfied: (a) the social composition function is linear and the marginal costs of contribution for contributors are identical, (b) the best response of each country is completely price inelastic and the marginal costs are identical, (2) a matching grant bring more benefits not only to the donor but also to the countries excluded from the policy than a lump-sum transfer but less benefits to the recipient, (3) a decline in the price of the public good for a country benefits the country if the substitution elasticity of the social composition function is smaller than one.

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