

Abstract

We evaluate the required size of a possible proof of Arrow's impossibility theorem. We consider the simplest case with two individuals, three social alternatives, and linear orderings for individual and social preferences. We formulate Arrow's theorem in propositional classical logic in the Gentzen-style proof theory. The size of a proof is measured by the number of leaves of a proof tree. We show that a proof is necessarily gigantic; a lower bound is 6^{36} and an upper bound $6^{37} + 420$. These numbers exceed the limit of human manageability to construct such a proof. Nevertheless, we have a proof of Arrow's theorem. The key concept distinguishing between the propositional calculus and our non-formalized proof is the use of "free variables". We consider this result from various points of views such as from that of limited inferences for game theoretical decision making, from that of logic and from that of social choice theory.