

Quasi-decisiveness, Quasi-ultrafilter, and Social Quasi-orderings

Susumu Cato*

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Abstract

This paper develops a new approach to the analysis of the underlying power structure when social preferences are allowed to be incomplete. We propose the concept of *quasi-decisiveness*, which is weaker than standard decisiveness, and investigate the family of quasi-decisive groups associated an Arrovian collective choice rule. A group is *quasi-decisive* if for every pair x, y of alternatives, y is not socially preferable to x in the weak sense if everyone in the group strictly prefers x to y . If social preferences are transitive, the family of quasi-decisive groups satisfies the following properties: (i) the group including all individuals is quasi-definitive, and the empty set is not quasi-decisive; (ii) adding individuals quasi-decisive group yields another quasi-decisive group; (iii) for two groups that are not quasi-decisive, the union of two groups is not quasi-decisive; (iv) if a group is not quasi-decisive, its complement is quasi-decisive. These properties except for (iii) are the same as those of an ultrafilter.

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*Tel: +81-857-29-1383, susumu.cato@gmail.com, Institute of Social Science, the University of Tokyo, 7-3-1, Hongo, Bunkyo-ku, Tokyo 113-0033, Japan.