## On the Existence of a Submodular Utility Function Koji Shirai April 10, 2010

## Abstract

This paper shows that, on lattices in the nonnegative integer lattices, every complete, weakly increasing, and quasisubmodular preordering has a submodular representation. This contrasts with the known result that every weakly increasing and quasisupermodular binary relation admits a supermodular representation on finite lattices. In addition, we show that, not only submodularity, the discrete version of the law of diminishing marginal utility is also non-refutable. By applying this result, we can show the discrete version of Afriat's theorem: the decreasing increment of a utility function cannot be refuted from the finite price-consumption data.

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