Generic Impossibility of Arrow's Impossibility Theorem

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Abstract

Bossert and Suzumura (2009) showed that the assignment of a quasi-transitive Arrovian collective choice rule F (not necessary reflexive and complete) to the corresponding set of decisive coalitions $\mathcal{V}_F$ defines a surjective map $\rho : \mathbb{CR}^{QT} \to \mathbb{F} \subseteq 2^{\mathcal{T}}$, where $\mathbb{CR}^{QT}$ is the set of all quasi-transitive Arrovian collective choice rules and $\mathbb{F}$ the set of all filters in $\mathcal{T}$. One major objective in the present paper is to determine the inverse image of the set of all ultrafilters $\mathbb{UF} \subseteq \mathbb{F}$ under ρ to be $\rho^{-1}(\mathbb{UF}) = \mathbb{CR}^{QT,SP}$, that is, the subset of $\mathbb{CR}^{QT}$ consisting of those satisfying the so-called strong preference property, which is also precisely the set of all Arrovian collective choice rules lying within $\mathbb{CR}^{QT}$ that admit dictators. Another major objective is to show that in the presence of infinitely many alternatives the set of Arrovian collective choice rules which fall into Arrow's impossibility theorem is "negligible" in the totality of quasi-transitive Arrovian collective choice rules, i.e., $\mathbb{CR}^{QT,SP}$ is nowhere dense in $\mathbb{CR}^{QT}$, where relevant spaces are equipped with suitable topologies.

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