Learning about one's own type in two-sided search

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Abstract

This paper is an analysis of a two-sided search model in which agents are vertically heterogeneous and some agents do not know their own types. Agents who do not know their own types update their beliefs about their own types through the offers or rejections that they receive from others. In the belief-updating process, an agent who is unsure of his or her own type frequently behaves as an over- or underconfident agent. In this paper, we show that this apparent over- or underconfidence influences both on the individual's and other agents' matching behaviors. We show, especially, that the apparent overconfidence of some agents prevents the lowest-type agents from matching in an equilibrium.

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Key Words: two-sided search; imperfect self-knowledge; overconfidence; looking-glass self

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1 Introduction

The "looking-glass self" has been the dominant concept in sociology and social psychology for the development of the self. This idea, attributed to Cooley (1902), is that people form their self-views by observing how others treat them. That is, others are significant as the "mirrors" that reflect images of the self. Although there is much literature on "looking-glass self" in the field of sociology and social psychology, the topic has received little attention in economics.[1]

In this paper, we introduce "looking-glass self" to a two-sided search model and study the implications of "looking-glass self" in the search behavior. We construct a model in which searchers do not know their own types, although they know the types of others. They then update their beliefs about their own types when they receive offers or rejections from others. For example, workers in search for an employer are evaluated by employers on their abilities when they meet. When a worker is young in terms of experience, her self-assessment is based on limited experience. On the other hand, employers may have considerable experience in assessing workers. At this time, when a young worker observes an offer or rejection from an employer, she infers something about her own type. Of course, when an experienced worker searches for a new job that is very similar to her previous job, she may have a more accurate self-view of her ability than employers. However, such situations are not considered in this paper. The key feature of our model is that others have better information than the agent. Similarly, in the search for a marriage partner, since a single agent is evaluated with regard to his or her marital charms by a member of the opposite sex when they meet, the individual of the opposite sex may have better assessments of the agents' charm than the agents themselves.[2] Hence, when an agent observes the offer or rejection from a member of the opposite sex, she infers something about her own type. In this paper, we show that this looking-glass self influences both their own and other agents' search behaviors.

We consider the basic framework of Burdett and Coles (1997), which is a two-sided search model with complete information. Their model can treat the marriage market, the labor market, the housing market, and other markets in which heterogeneous buyers and sellers search for the right trading partner.[3] Using the marriage market interpretation, the model is described as follows. Single agents are vertically heterogeneous, i.e., there exists a ranking of marital charm (types). For simplicity, we assume that there are three types of men/women according to charm: high, middle, and low. Single men/women enter the market in order to look for a marital partner. When a man and a woman meet, an opponent's type can be recognized. The agent's optimal search strategy has the reservation level property, i.e., he or she continues searching until he or she meets a member of the opposite sex that is at least

[1]As we discuss below, in economics, Bénabou and Tirole (2003), Ishida (2006), and Swank and Visser (2006) consider "the looking-glass self" in principal-agent models.

[2]Marital charm is defined by various elements, including quality, attraction, intelligence, height, age, education, and family background, in much of the literature regarding marriage.

[3]In the labor market, workers and employers seek each other as working partners. Moreover, both workers and firms are ex-ante heterogeneous: workers' productivity differs according to each individual's ability and skills, and a firm's productivity also differs according to its capital holdings.

as good as the predetermined threshold, called the "reservation level," which depends on the agent's search cost and the type distribution of agents. If a man and a woman meet and both agents propose, they marry and leave the market. If at least one of the two decides not to propose, they separate and continue to search for another partner. Given these settings, the marriage pattern—who marries whom—in the market is determined. This marriage pattern becomes a kind of positive assortative matching.[4]

When an agent (she) is unsure of her own type, she behaves as an over- or underconfident agent often in her belief-updating process. The woman with imperfect self-knowledge may reject a man whom the woman with perfect self-knowledge accepts. Therefore, this woman will apparently overestimate her actual type. Since this *apparent overconfidence* is generated due to the correct belief-updating process, agents in this study are fully unbiased and rational in the sense that they have no false information and follow Bayes's rule in updating their posterior beliefs about their own types.[5] Likewise, we use the term *apparent underconfidence* if a woman with imperfect self-knowledge accepts a man whom she rejects when she knows her own type.

We show that the apparent over- or underconfidence in the belief-updating process generates two externalities: the first is *direct externality*: the rejection of an apparently overconfident woman delays the timing of marriage of the man who is directly rejected by her when they meet. On the other hand, the acceptance of an apparently underconfident woman makes the future partner better off, as she increases the value of the match to the partner. If there are many apparently overconfident women in the market, the second externality is generated: the men who are now rejected by the apparently overconfident women accept another lower type of women who are rejected by these men when all agents know their own types. We call this change in an agent's behavior due to the apparent over- or underconfidence of other agents *indirect externality.* Moreover, in a two-sided search framework, the women who are now accepted by these men may also reject the men whom these women accept when all agents know their own types. Then, the indirect externality may spread across the market. However, this indirect externality is generated only by an apparent overconfidence. We show that the apparent underconfidence does not have the indirect externality.

We obtain the following results: first, we derive the conditions under which the economy is at a *perfect sorting equilibrium* in which only persons of the same type marry if all agents know their own types as a benchmark case.

Secondly, we investigate the case of a perfect sorting equilibrium with women's imperfect self-knowledge (even if sex is reversed, the following results are confirmed). In this case, since some middle-type women who are unsure of their own types reject middle-type men, middle-type men lower their reservation levels for low-type women compared with the benchmark

[4]Positive assortative matching is said to hold if the characteristics (types and marital charm) of those who match are positively correlated. Becker (1973) found strong empirical evidence of a positive correlation between the characteristics of partners.

[5]In contrast, 'overconfidence' is generally generated due to some error in an agent's processing information. For example, overconfidence would occur, when an agent overestimates the type distributions in the market or when an agent selects information in her belief updating process.

case. On the other hand, women with imperfect self-knowledge lower or raise their reservation levels for the men relative to the benchmark case, as they assign probabilities to their own types.

Thirdly, we consider the apparent overconfidence case in which many middle-type women raise their reservation levels to reject middle-type men due to imperfect self-knowledge of these middle-type women. Since there are sufficient large numbers of apparently overconfident middle-type women, middle-type men change their behavior: they accept proposals from low-type women. In a two-sided search, this indirect externality of apparent overconfidence further makes low-type women with *perfect* self-knowledge change their reservation strategies: given the behavior of middle-type men, low-type women who know their own types reject low-type men. As a result, low-type men cannot marry. That is, the indirect externality of apparent overconfidence spreads to lower-type agents and then prevents the lowest-type men from marrying.

Finally, we consider the case of apparent underconfidence, in which many middle-type women lower their reservation levels to accept low-type men due to imperfect self-knowledge of these women. In this case, if low-type men accept middle-type women and reject low-type women, the offer from a low-type man for the apparently underconfident middle-type woman informs her that she is not a low-type woman. At this time, this middle-type woman has the incentive to reject a low-type man. Therefore, apparent underconfidence does not have indirect externality. That is, a low-type man always accepts a low-type woman even if there are many middle-type women who accept low-type men. As a result, there are no agents who cannot marry in the case of apparent underconfidence.[6]

Our results show that, when there are agents with an imperfect self-knowledge under the cloning assumption and the non-transferable utility, multiple equilibria can arise in some parameter ranges: the equilibrium in the perfect sorting equilibrium with imperfect self-knowledge and the equilibrium in the apparent underconfidence case hold in some parameter ranges. By contrast, when all agents know their own types under the cloning assumption and the assumption of non-transferable utility, a unique equilibrium always arises (see, Burdett and Coles (1997)). In our multiple equilibrium case, marriage patterns are determined by the expectation of all agents about the behavior of agents with imperfect self-knowledge.

The result in which the lowest-type agents cannot marry in an apparent overconfidence case is consistent with the recent data of educational assortative marriage patterns in the United States and in Japan. In the U.S. and Japan, the percentage of never married men/women increases and, in particular, that of never married men/women with a low level of education is notably high.[7] According to the U.S. Census Bureau data, in 2006, the percentage of never-married individuals at age 35-44 is 24% for men with high school

[6] In our analysis, a man and a woman are assumed to propose or reject a member of the opposite sex simultaneously. Similar results will also be obtained in the case of the sequential move in which a man proposes to a woman in the first move and she proposes/rejects him in the next move.

[7] Schwartz and Mare (2005) explain that the trend of educational assortative marriage in the U.S. is led by the declining economic standing of men with a low level of education from the late 1970s through the mid-1990s. Nosaka (2009) shows that an increase in the wages of highly educated women leads to an increase in the number of unmarried individuals among persons with low education in a marriage model.

education or less and 14% for women with high school education or less. On the other hand, the percentage of never-married individuals at age 35-44 is 14% for men with some college education or more and 12% for women with some college education or more. Moreover, in Japan, the decline in marriage has been most pronounced among less-educated men at age 35-39 (Raymo and Iwasawa 2005). Our results suggest that the apparent overconfidence accelerates the increase in the proportion of never-married men/women with a low level of education, since education is one of the elements of charm.

Related literature

Early psychologists and sociologists thought that the self was built on reflected assessments—people form their self-views by observing how others treat them. James (1890), who set the stage for the idea of "looking-glass self", argued that the self was a product and reflection of social life. The idea of the "looking-glass self" was introduced by Cooley (1902). He expanded the idea that the self develops by referencing other people in the social environment. Cooley maintained that the person observes how others view him- or herself and then incorporates those views into the self-view. Mead (1934) further developed the idea of Cooley (1902).[8] Following this long tradition, most researchers in psychology and sociology accepted that others are significant as the "mirrors" which construct and modify the self-view (for example, Goffman (1959), Baumeister (1982, 1986), Wicklund and Gollwitzer (1982), Gollwitzer (1986), Rhodewalt (1986), Schlenker (1986), Swann (1987, 1990, 1996), Cole (1991), Kenny and Depaulo (1993), Swann, De La Ronde, and Hixon (1994), Bartusch and Matsueda (1996), Kelly (2000), Tice and Wallace (2003)).[9]

In Economics, recent work has introduced the idea of "looking-glass self" (for example, Bénabou and Tirole (2003), Ishida (2006), and Swank and Visser (2006)). Bénabou and Tirole (2003), who presented the principal-agent model, assume that, whereas the principal knows the agent's type, an agent has imperfect knowledge about his own type. As the principal prefers to offer the bonus when facing an agent with low ability, a high bonus becomes the signal that the principal does not trust the agent. Thus, a high bonus diminishes the agent's self-confidence. In contrast, giving a challenging task increases the agent's self-confidence. Given that effort and ability are usually complements, the more confidence the agent has with regard to his ability, the more effort he exerts. Swank and Visser (2006) focus on the quality of the principal's information about the agent's type and show that the quality of the principal's information determines whether or not a delegation can be used as a means of communicating this information to an agent. Ishida (2006) applies the framework of Bénabou

[8]In his view, people are affected not only by how they think significant others respond to them but also by how they think their entire social group does.

[9]A body of literature supports the looking-glass self theory with respect to the evaluative quality of the self's attributes. Pinhey, Rubinstein, and Colfax (1997) found that overweight people were not significantly happy in cultures in which thinness was valued (see also Ross (1994) and Cioffi (2000)). Ichiyama (1993) conducted an experiment and showed that others' actual assessments of interpersonal behavior were linked to self-assessments and reflected assessments (what participants think others think of them). Furthermore, Jussim, Soffin, Brown, Ley, and Kohlhepp (1992) conducted several experiments and found that, when people were fully aware of how others viewed them, their self-perceptions were indeed affected by reflected assessments.

and Tirole (2003) to promotion policies. He endogenizes the degree of information asymmetry and derives dynamic implications. In contrast with these studies, we apply the idea of the "looking-glass self" to the two-sided search model and not to the principal-agent model.

In our model, some women with imperfect self-knowledge behave as over- or underconfident agents. Our paper is then related to the studies of over- or underconfidence. A study by Dubra (2004) is the most closely related one to ours; Dubra examines the implications of overconfidence in the search behavior of workers. The difference from our model is that his is a one-sided search and an overconfident worker has false prior belief (then, overconfident workers in his model overestimate their chances of finding a better offer). In this paper, although apparently overconfident agents do not know their own types, they do not have false prior belief, i.e., they have correct expectations about the type distributions in the market. Moreover, the two-sided problem generates the indirect externality of apparent overconfidence.

There are several recent studies about overconfidence of workers in addition to the study by Bénabou and Tirole (2003) and Dubra (2004) mentioned earlier. Camerer and Lovallo (1999) show that overconfidence about relative ability leads to excessive business entry by creating experimental entry games. Santos-Pinto and Sobel (2005) show that a subjective positive/negative self-image arises when different people have different opinions about how skills determine ability.[10] Furthermore, Benoît and Dubra (2008) show that, even if everyone perfectly understands the level of skills in the population, they are apparently overconfident in a signaling model. Our agents are similar to agents in their model in the sense that they are rational, have correct information, and are Bayesian. However, we examine the influence of this rational belief-updating process on agents' search behavior.

This paper is organized as follows. Section 2 is a description of the basic framework for our analysis. In Section 3, we show the consequence of the benchmark case, in which all agents know their own types. Next, we examine the case of a perfect sorting equilibrium with imperfect self-knowledge. Thirdly, we investigate the apparent overconfidence case, in which middle-type women with imperfect knowledge about their own types reject middle-type men whom middle-type women with perfect self-knowledge accept. Finally, we examine an apparent underconfidence case in which middle-type women accept low-type men because of the uncertainty of these women's own types. In Section 4, we analyze the number of marriages and social welfare generated by marriages. In Section 5, we discuss the extensions of the model. Section 6 is the conclusion.

[10] Although their study mainly analyzes the positive/negative self-image in a skill acquisition model, we can apply the same logic to our marriage market model. In such a case, individuals must invest in each component of charm to maximize their own charm before they enter the market. This assumption follows the idea of that proposed by Burdett and Coles (2001), in which individuals increase their charm before they participate in the marriage market. Moreover, assuming that some individuals disagree about the contribution of each element of charm to effective charm, they overestimate or underestimate their own charm.

2 The basic framework

In this section, we present a basic framework for our analysis in this study. Throughout, we restrict our attention to the steady state.

Let us assume that there are a large and equal number of men and women in a marriage market. Let N denote the participating men/women in this market. An agent in the market wishes to marry a member of the opposite sex.

Finding a marriage partner always involves a time cost. It is difficult for agents to meet someone of the opposite sex in the market. Let α denote the rate at which a single individual contacts a member of the opposite sex, where α is the parameter of the Poisson process.

It is assumed that agents are ex-ante heterogeneous and all agents have the same ranking about a potential partner in the marriage market. Let x_k denote the type (charm) of a single man/woman k in the market; it is assumed to be a real number.

When both sexes meet, each agent can instantly recognize the opponent's type x_k and then decide whether or not to propose. We assume that both agents submit their offers or rejections simultaneously.[11] Therefore, if at least one of the two decides not to propose, they return to the marriage market and search for another partner. If both agents propose, they would marry and leave the marriage market permanently. We assume that, if a couple marries, he or she obtains a utility flow equal to the spouse's type per unit of time and vice versa. That is, utilities are *non-transferable*: there is no bargaining for the division of the total marital utility. Let us assume that people live forever and there is no divorce.

Let us assume that x_k is drawn from $F_i(x)$, $i = m, w$, which denotes the distribution of actual types among men (m)/women (w) in the market. Although $F_m(x)$ and $F_w(x)$ need not be symmetric among men and women, for simplicity, let us assume that $F_m(x)$ and $F_w(x)$ are symmetric among men and women. All agents know $F_m(x)$ and $F_w(x)$.

An equilibrium is a steady state in which all agents maximize their expected discounted utilities given that they have correct expectations about the strategies of all others in the market. A steady state requires that the exit rate of each type equals the entry rate of new agents of that type. To simplify the analysis, we assume that, if a pair marries and leaves the market, two identical agents enter the market at once (see, for example, MacNamara and Collins (1990), Morgan (1994), Burdett and Coles (2001), Bloch and Ryder (2000), and Cornelius (2003)).[12]

[11] In our analysis, a man and a woman are assumed to propose/reject a member of the opposite sex simultaneously. Similar results can be also obtained in the case of a sequential move in which a man proposes to a woman in the first move and she proposes/rejects him in the next move.

[12] Some other assumptions of "inflow" have been considered in Burdett and Coles (1999). With other reasonable assumptions of inflow, such as exogenous inflow, the analysis is very complicated in our framework with learners. Moreover, now, our attention is focused not on the type distribution, which is derived under an assumption of inflow, but on the marriage pattern (i.e., who marries whom) in a steady state when there are agents with imperfect self-knowledge. We then apply the "cloning assumption" to our model for technical simplicity.

3 Analysis

In this section, first, to show the externality of belief-updating process, we derive the conditions under which the economy is at a perfect sorting equilibrium, in which only persons of the same type marry, if all agents know their own types as a benchmark. In later subsections, we study three cases with *imperfect self-knowledge* (i.e., agents do not know their own types perfectly) and compare these three cases with the benchmark case.

3.1 Perfect self-knowledge–Benchmark result

In this subsection, we consider the *perfect self-knowledge* case in which all agents know their own types. To simplify the analysis, let us assume that there are three types of men/women according to charm: high (H), middle (M), and low (L).[13] A participant in a marriage market belongs to one of these types. Let x_H/r denote the (discounted) utility of marrying a high-type agent; similarly, x_M/r and x_L/r represent the utilities of marrying a middle-type agent and a low-type agent, respectively, where $r > 0$ is the discount rate. We assume that $x_H > x_M > x_L > 0$. That is, in any equilibrium, all agents would like to marry a high-type agent. Both sexes are assumed to obtain zero utility flow while they are single.

Let λ_H^i $(i = m, w)$ denote the proportion of high-type men/women in the marriage market. Similarly, λ_M^i and λ_L^i are the proportion of men/women who belong to the middle and low types, respectively, where $\lambda_H^i + \lambda_M^i + \lambda_L^i = 1$. Although λ_k^i $(k = H, M, L)$ of each sex need not be symmetric among men and women, for simplicity, let us assume that λ_k^i $(k = H, M, L)$ of each sex are symmetric. We will use subscript i $(= m, w)$ to indicate men/women for the explanation of the results.

First, we consider the decision of a high-type man. He decides whether to accept or reject a woman of the middle- or low-type. The expected discounted lifetime utility of a single high-type man V_H becomes

$$\begin{aligned} rV_H = {} & \alpha\lambda_H^w \left(\frac{x_H}{r} - V_H\right) \\ & + \alpha\lambda_M^w \left[\max\left(V_H, \frac{x_M}{r}\right) - V_H\right] + \alpha\lambda_L^w \left[\max\left(V_H, \frac{x_L}{r}\right) - V_H\right]. \end{aligned} \tag{1}$$

A high-type man ($k = H$) meets a high-type woman, and they marry with probability $\alpha\lambda_H^w$. However, if a high-type man meets a middle- (low-) type woman with probability $\alpha\lambda_M^w$ ($\alpha\lambda_L^w$), he compares x_M/r (x_L/r) and V_H and then decides whether or not to propose. By this comparison, we can obtain the optimal strategy of the high-type man, given α and $F_i(x)$. This optimal strategy has the feature of the reservation utility level for rejecting other types; that is, a k-type man will accept any offer $x_{k'} \geq R_k$ from a k'-type woman, where $R_k = rV_k$. Likewise, we obtain the optimal reservation strategies of each type of men/women.

[13] As we will discuss in detail in Section 5, we assume not two but three types of agents in order to show the indirect effect (indirect externality) of the belief-updating process: even if the agents with perfect self-knowledge do not directly meet agents with imperfect self-knowledge, these agents with perfect knowledge may change their marriage behavior more than those with perfect self-knowledge.

We restrict our attention to the next equilibrium in this article in order to show the influences of the externality of the belief-updating process on a marriage market.

Definition 1 *In* the perfect sorting equilibrium (PSE)*, high-type agents marry within their group, as do middle-type agents and low-type agents.*

In the PSE, men and women of the same type marry. Therefore, we can consider that high-type agents who marry within their group form the first cluster of marriages, middle-type agents who marry within their group form the second cluster of marriages, and low-type agents who marry within their group form the third cluster of marriages in this equilibrium. We now define the following situation as a benchmark case: if all agents know their own types, the PSE occurs. The following proposition shows the condition for the PSE.

Proposition 1 *(PSE) Let us assume that all agents recognize their own types. The economy is at the PSE if*

$$x_M < R_H^* \equiv \frac{\alpha\lambda_H^i x_H}{\alpha\lambda_H^i + r}, \quad i = m, w, \tag{2}$$

and

$$x_L < R_M^* \equiv \frac{\alpha\lambda_M^i x_M}{\alpha\lambda_M^i + r}, \quad i = m, w. \tag{3}$$

Proof. If a high-type agent turns down a middle-type agent of the opposite sex agent i $(= m, w)$, $V_H > \frac{x_M}{r}$. From (1), this high-type agent's discounted lifetime utility when he or she is single becomes

$$rV_H^r = \alpha\lambda_H^i \left(\frac{x_H}{r} - V_H^r\right).$$

On the other hand, when he or she accepts a middle-type agent i and turns down a low-type agent i, i.e., $\frac{x_M}{r} \geq V_H > \frac{x_L}{r}$, his/her value function is[14]

$$rV_H^a = \alpha\lambda_H^i \left(\frac{x_H}{r} - V_H^a\right) + \alpha\lambda_M^i \left(\frac{x_M}{r} - V_H^a\right).$$

If $V_H^r > V_H^a$ is satisfied, a high-type agent refuses a middle-type opposite sex agent i. This inequality $V_H^r > V_H^a$ means that

$$x_M < R_H^* \equiv \frac{\alpha\lambda_H^i x_H}{\alpha\lambda_H^i + r}.$$

If $x_M \geq R_H^*$, a high-type agent proposes to a middle-type agent i.

Under inequality (2), we can obtain the condition for a middle-type agent to reject a low-type opposite sex agent i by the same process as that described above. Consequently, we

[14]If a high-type agent proposes to a middle-type agent but turns down a low-type agent ($x_L/r < V_H \leq x_M/r$), the high- and middle-type agents receive at least the same number of offers. Hence, $V_H \geq V_M$, and then we have $V_M \leq x_M/r$. Namely, a middle-type agent wishes to marry another middle-type agent. Likewise, if a middle-type agent accepts a low-type agent ($V_M \leq x_L/r$), the middle- and low-type agents receive at least the same number of offers. Then, a low-type agent also wants to marry another low-type agent.

have

$$x_L < R_M^* \equiv \frac{\alpha\lambda_M^i x_M}{\alpha\lambda_M^i + r}.$$

■

The parameter $\alpha\lambda_H^i$ $(i = m, w)$ implies the arrival rate of proposals for an agent to contact a high-type opposite sex agent i. Similarly, $\alpha\lambda_M^i$ $(\alpha\lambda_L^i)$ is the rate at which an agent meets a middle- (low-) type opposite sex agent i. The inequality (2) can be rewritten as the condition of parameter λ_H^i. Therefore, Proposition 1 means that, with constant α, if λ_H^i is large enough $(\alpha\lambda_H^i > \frac{rx_M}{(x_H - x_M)})$, a high-type agent turns down a middle- and a low-type opposite sex agent i in the market $(x_M < R_H^*)$. Conversely, if there are sufficiently few high-type opposite sex agent i $(\alpha\lambda_H^i \leq \frac{rx_M}{(x_H - x_M)})$, a high-type agent will accept a middle-type opposite sex agent i $(x_M \geq R_H^*)$. If $\alpha\lambda_H^i \leq \frac{rx_L}{(x_H - x_L)}$ holds, a high-type agent is willing to propose to any agent of the opposite sex $(x_L \geq R_H^*)$. A similar discussion can be done for parameter λ_M^i by inequality (3). If $x_M < R_H^*$ and $x_L < R_M^*$ are satisfied, $V_H > x_M/r > V_M > x_L/r > V_L$ holds. If λ_H^i and λ_M^i are small enough to satisfy $\alpha\lambda_H^i \leq \frac{rx_M}{(x_H - x_M)}$ and $\alpha\lambda_M^i \leq \frac{rx_L - \alpha\lambda_H^i (x_H - x_L)}{(x_M - x_L)}$, all agents obtain the same expected discounted lifetime utility: $V_L = V_M = V_H < \frac{x_L}{r}$. In this case, all types accept each other, and then all agents marry the first agent of the opposite sex that they meet.[15]

It is noteworthy that, if $r = 0$, then $x_M < R_H^*$ and $x_L < R_M^*$ hold. Therefore, the equilibrium is the PSE when $r = 0$.

In the following subsections, we introduce the imperfect knowledge about agents' own types into the benchmark case. To investigate the influence of externality of the belief-updating process, in the following subsections, we consider the case in which the condition in Proposition 1 is satisfied: $x_M < R_H^*$ and $x_L < R_M^*$ hold.[16]

3.2 Imperfect self-knowledge

In this subsection, we introduce the imperfect self-knowledge into the benchmark case. In an agent's belief-updating process, she/he will use a different strategy from that in the benchmark case. We then consider the following three cases in this subsection. First, we consider the case of PSE with imperfect self-knowledge, in which, even if there are some agents who do not know their own types, only persons of the same type marry. Next, we consider *the case of apparent overconfidence*, in which some women with imperfect self-knowledge accept the men whom they reject when they know their own types. Finally, we consider *the case*

[15] Four possible steady-state equilibrium outcomes can be considered when all agents have the perfect self-knowledge: Equilibrium (i) agents of the same type marry (PSE); Equilibrium (ii) agents of the high-type and the middle-type form the first cluster of marriages, and agents of the low-type form the second cluster of marriages; Equilibrium (iii) agents of the high-type form the first cluster of marriages, and agents of the middle- and low-type form the second cluster of marriages; and Equilibrium (iv) all agents marry the first person of the opposite sex they meet (they form one cluster of marriage). From Proposition 1, equilibrium (ii) occurs when $R_H^* \leq x_M,\ R_M^* > x_L$. Equilibrium (iii) occurs when $R_H^* > x_M,\ R_M^* \leq x_L$.Equilibrium (iv) occurs when $R_H^* \leq x_M,\ R_M^* \leq x_L$.

[16] For other parameter ranges, it is difficult to show the indirect effect (indirect externality) of the belief-updating process. We will discuss it in detail in Section 5.

of apparent underconfidence, in which some women with imperfect self-knowledge reject the men whom they accept when they know their own types.

Let us assume that all agents understand the type distribution $F_m(x)$ and $F_w(x)$ and that all men know their own types. However, all women do not initially know their own types when they have just entered the marriage market.[17] We refer to them as 'k_0-type' women, where k represents their actual type, i.e., $k = H, M, L$. Therefore, a woman who does not know her own type has a belief about her own type.

Women with imperfect self-knowledge may learn something about their actual types after a meeting with a man. For example, consider that a high-type man accepts only a high-type woman and this is common knowledge among all agents. If a woman is rejected by him, she then learns that she is either a middle- or a low-type following a meeting with him. Therefore, a woman's belief about her own type depends on men whom she met in the past. As a result, there are different kinds of women with different beliefs even if they belong to the same actual type. Let $G_m(x)$ and $G_w(x)$ denote the distribution of men with respect to differences in their beliefs and that of women according to differences in their beliefs, respectively. Let us assume that all agents also know $G_m(x)$ and $G_w(x)$. (We show later that $G_m(x)$ and $G_w(x)$ depend on α and $F_i(x)$, which are common knowledge among all agents.) Therefore, all agents choose optimal strategies using $G_m(x)$ and $G_w(x)$. Moreover, let us assume that any woman's prior belief about her own type is $G_w(x)$. It is noteworthy that $G_m(x) = F_m(x)$ as all men know their own types.

Let us assume that a woman updates the belief about her own type by Bayes's rule, when the offer or rejection from a man carries information about her actual type. For instance, consider that a high-type man accepts only a high-type woman and this is common knowledge among all agents. We can express the posterior belief of the k_0-type ($k = H, M, L$) woman who met him as

$$\begin{aligned}
&\Pr(\text{H}|\text{reject from H}) \\
&= \frac{\Pr(\text{H})\Pr(\text{rejct from H}|\text{H})}{\Pr(\text{H})\Pr(\text{rejct from H}|\text{H})+\Pr(\text{M})\Pr(\text{rejct from H}|\text{M})+\Pr(\text{L})\Pr(\text{rejct from H}|\text{L})} = 0, \\
&\Pr(\text{M}|\text{reject from H}) \\
&= \frac{\Pr(\text{M})\Pr(\text{rejct from H}|\text{M})}{\Pr(\text{H})\Pr(\text{rejct from H}|\text{H})+\Pr(\text{M})\Pr(\text{rejct from H}|\text{M})+\Pr(\text{L})\Pr(\text{rejct from H}|\text{L})} = \frac{\lambda'_M}{\lambda'_M+\lambda'_L}, \\
&\Pr(\text{L}|\text{reject from H}) \\
&= \frac{\Pr(\text{L})\Pr(\text{rejct from H}|\text{L})}{\Pr(\text{H})\Pr(\text{rejct from H}|\text{H})+\Pr(\text{M})\Pr(\text{rejct from H}|\text{M})+\Pr(\text{L})\Pr(\text{rejct from H}|\text{L})} = \frac{\lambda'_L}{\lambda'_M+\lambda'_L},
\end{aligned}$$

where $\Pr(\text{M}) = \lambda'_M$ and $\Pr(\text{L}) = \lambda'_L$. Her optimal strategy is changed by using this belief after she updates her belief.

[17] In a one-sided imperfect knowledge assumption, it is important to clarify the influences of imperfect self-knowledge. We discuss this in detail in Section 5. This one-sided imperfect knowledge assumption describes the following situations: in the context of the labor market, a firm will have more information than a worker about its own type, since a firm generally has more experience than a worker. In the context of the marriage market, this assumption describes a situation in which it would be easier for men to obtain objective data on their own charm, such as income, position at work, and social status, than for women in cultures in which more men work outside the home than women.

3.2.1 PSE with imperfect self-knowledge

In this subsubsection, we find the PSE with imperfect self-knowledge (we call this equilibrium 'PSEI'). In the PSEI, k_0-type ($k = H, M, L$) women always reject middle-type men. Otherwise, the PSEI does not occur as men and women of the different type marry.[18]

First, we investigate the optimal strategies of men when there are women with imperfect self-knowledge. A man decides his optimal strategy given women's *behavior*—who accepts (or rejects) whom—in the market. We will use the term "behavior" to distinguish it from "strategy" in the following sections. In our model with discrete types, even if an agent lowers his reservation utility strategy, this does not guarantee that he accepts an agent whom he has rejected previously. Therefore, the sentence that an agent changes his *behavior* means that he changes the type of woman whom he accepts (or rejects). The strategy (and behavior) of each type is common knowledge among all agents, as all agents know $G_i(x)$, $i = m, w$.

As we consider the PSEI, all high-type women (including high-type women with imperfect self-knowledge) always reject middle-type men, all middle-type women (including middle-type women with imperfect self-knowledge) always reject low-type men, and some low-type women (at least, low-type women with perfect self-knowledge) accept low-type men.

Given the behavior of these women, all men decide their optimal strategies. A high-type man has the same reservation utility as a high-type man in a PSE since all women want to marry high-type men. The decision of a middle-type man is as follows. Now, there are always some middle-type women who reject middle-type men, as an M_0-type woman rejects a middle-type man at least. On the other hand, there are some middle-type women who accept middle-type men, as a high-type man rejects a middle-type woman.[19] Let $\eta \in (0,1)$ denote the proportion of middle-type women who accept middle-type men. Similarly, let $\zeta \in (0,1)$ denote the proportion of low-type women who accept middle-type men, as there are always some low-type women (including L_0-type women) who reject middle-type men. The following Lemma applies to a middle-type man.

Lemma 1 *Let us assume that $R_H^* > x_M$, $R_M^* > x_L$, and that $\eta \in (0,1)$ of middle-type women accept middle-type men and $\zeta \in (0,1)$ of low-type women accept middle-type men. If*

$$x_L < (\geq)\ R_{Mm}^p \equiv \frac{\alpha\eta\lambda_M^w x_M}{\left(r+\alpha\eta\lambda_M^w\right)}, \tag{4}$$

a middle-type man rejects (accepts) a low-type woman. In this case, the reservation level of a middle-type man for a low-type woman decreases, in contrast with the benchmark result, i.e., $R_M^* > R_{Mm}^p$.

Proof. See, Appendix A. ■

This lemma means that the rejections of some middle-type women for middle-type men lower the reservation utility level of a middle-type man for a low-type woman.

[18]Moreover, all agents always reject agents of a lower type than themselves in a PSEI.

[19]If a woman who was rejected by a high-type man turns down a middle-type man, her expected discounted lifetime utility becomes zero.

We can rewrite $R^p_{Mm} > (\leq)\ x_L$ as $\alpha\eta\lambda^w_M > (\leq)\ \frac{rx_L}{(x_M - x_L)}$. This means that, with constant α, if $\eta\lambda^w_M$ is small enough ($\alpha\eta\lambda^w_M \leq \frac{rx_L}{(x_M - x_L)}$), a middle-type man accepts a low-type woman. Conversely, if there are enough middle-type women who accept middle-type men ($\alpha\eta\lambda^w_M > \frac{rx_L}{(x_M - x_L)}$), a middle-type man could turn down a low-type woman due to his expectation to marry a middle-type woman who accepts him. Let us assume that $R^p_{Mm} > x_L$ in this subsubsection, in order to focus on the PSEI. Given $R^p_{Mm} > x_L$, a low-type man has the same behavior as one in the PSE: he accepts a low-type woman.

When $R^p_{Mm} > x_L$, the middle-type man's rate of contact with the woman whom he wishes to marry is $\alpha\eta\lambda^w_M$. Then, a middle-type man's time (duration) until meeting such a woman is $\frac{1}{\alpha\eta\lambda^w_M}$. Therefore, his time until marriage is delayed due to the rejection of middle-type women with imperfect self-knowledge, since that in the benchmark case is $\frac{1}{\alpha\lambda^w_M}$. This delay of marriage is the *direct* (negative) *externality* of the imperfect self-knowledge.

Next, we investigate the optimal strategies of women. A woman decides her optimal strategy given men's behavior (i.e., 'who accepts whom'). In the PSEI, high-type men reject middle-type women ($R^*_H > x_M$), middle-type men reject low-type women ($R^p_{Mm} > x_L$), and low-type men accept low-type women. Let us confirm the information from rejections or acceptances by men in order to obtain the strategies of women. Now, a high-type man rejects a middle-type woman. Thus, after a woman is rejected by a high-type man, she learns that she is either the middle- or the low-type. In that way, she updates her belief about her type.[20] Conversely, if a woman receives a proposal by a high-type man, she marries him and learns that she is a high-type. Likewise, since a middle-type man accepts a middle-type woman in the PSEI, the proposal by a middle-type man means that the woman who receives a proposal from him is a high- or middle-type. On the other hand, a rejection by the middle-type man means that the rejected woman is a low-type. Since a low-type man proposes to any woman in the PSEI, the women who receives his proposal learns nothing about her own type.

Given men's behavior, we can consider women's belief-updating process. Figure 1 contains a description of it. The outline box for each type in Figure 1 represents the proportion of each type of women $\lambda^w_k, (k = H, M, L)$. First, we consider the belief-updating process of high-type women. If an H_0-type woman meets a high-type man who accepts a high-type woman, then she learns that she is a high-type, leaving the market with him. When another H_0-type woman meets a middle-type man, she always rejects him in the PSEI. However, she learns that she is the high- or the middle-type as she can know that she received the offer by that middle-type man. We refer to the woman who learns that she is either a high- or middle-type as a 'H_{HM}-type' woman. The subscript represents the possible types of a woman. Now, we assume that an H_{HM}-type woman rejects a middle-type man (indeed, as is shown below, she always rejects a middle-type man in the PSEI). Hence, if an H_{HM}-type woman meets a high-type man, she leaves the market with him and learns that she is a high-type. As a result, there are two kinds of high-type women (H_0-type and H_{HM}-type) in the market. Here, let $\theta \in (0, 1)$ denotes the proportion of H_{HM}-type women.

Likewise, we can consider the belief-updating process of middle-type and low-type women

[20] She then changes her behavior. She accepts an offer from a middle-type man at the lowest.

(see Figure 1).[21] Consequently, there are four kinds of middle-type women. Here, let $\mu \in (0,1)$, $\gamma(1-\phi) \in (0,1)$ and $\gamma\phi \in (0,1)$ denote the proportion of M_{ML}-type women, M_{HM}-type women, and M_M-type women, respectively. For the low-type women, there are four kinds of low-type women.[22] Here, let $\psi(1-\nu) \in (0,1)$, $\psi\nu \in (0,1)$, and $\kappa \in (0,1)$ denote the proportion of L_{ML}-type women, L_{L1}-type women, and L_{L2}-type women, respectively.

From these proportions $(\theta, \mu, \gamma, \phi, \psi, v, k)$, the distribution of women with respect to their belief differences $G_w(x)$ is given.[23]

Under these settings, the optimal strategies of an l_{ML}-type $(l = M, L)$, j_{HM}-type $(j = H, M)$, and k_0-type $(k = H, M, L)$ woman are obtained in the next lemma.

Lemma 2 *Let us assume that a high-type man accepts a high-type woman ($R_H^* > x_M$), a middle-type man accepts a middle-type woman $\left((R_M^* >)\ R_{Mm}^p > x_L\right)$, and a low-type man accepts a low-type woman. If*

$$x_L < (\geq) \frac{\alpha\mu\lambda_M^w\lambda_M^m x_M\left(r+\alpha\lambda_L^m\right)}{\left(\mu\lambda_M^w\left(r+\alpha\lambda_L^m\right)\left(r+\alpha\lambda_M^m\right)+\lambda_L^w r\psi(1-\nu)\left(r+\alpha\lambda_L^m+\alpha\lambda_M^m\right)\right)} \equiv R_{l_{ML}}^p, \tag{5}$$

an l_{ML}-type $(l = M, L)$ woman rejects (accepts) a low-type man, where $R_{l_{ML}}^p < R_M^*$. If

$$x_M < (\geq) \frac{\lambda_H^w\lambda_H^m\theta\alpha x_H\left(r+\alpha\lambda_M^m\right)}{\left(\lambda_H^w\theta\left(r+\alpha\lambda_H^m\right)\left(r+\alpha\lambda_M^m\right)+r\gamma(1-\phi)\lambda_M^w\left(r+\alpha\lambda_H^m+\alpha\lambda_M^m\right)\right)} \equiv R_{j_{HM}}^p = R_{k_0}^p, \tag{6}$$

a j_{HM}-type $(j = H, M)$ or a k_0-type $(k = H, M, L)$ woman rejects (accepts) a middle-type man, where $R_{j_{HM}}^p = R_{k_0}^p < R_H^*$.

Proof. See, Appendix A. ■

From this Lemma, women with imperfect self-knowledge assign probabilities to their own types. Therefore, the reservation utility levels of M_{ML}-type women, H_{HM}-type women, and H_0-type women are lowered, in contrast with the benchmark results. On the other hand, the reservation utility levels of L_{ML}-type women, M_{HM}-type women, and k_0-type women $(k = M, L)$ are raised, in contrast with the PSE.

When $r = 0$, $R_{l_{ML}}^p = R_M^* (= x_M)$ holds. Therefore, an l_{ML}-type woman always prefers to meet a middle-type man over accepting a low-type man in order to have the chance to

[21] A belief-updating process of middle-type women is as follows. An M_0-type woman learns that she is a high- or middle-type after she meets a middle-type man, who accepts a high- or middle-type woman in the PSEI. Then, she becomes a 'M_{HM}-type' woman. An M_{HM}-type woman rejects a middle-type man in the PSEI, similarly to a H_{HM}-type woman. Then, an M_{HM}-type woman further becomes a 'M_M-type' woman after a meeting with a high-type man, who rejects her. An M_M-type woman leaves the market with a middle-type man when they meet. On the other hand, another M_0-type woman becomes a 'M_{ML}-type' woman by meeting with a high-type man. An M_{ML}-type woman rejects a low-type man in the PSEI. (Otherwise, an equilibrium does not become a PSEI, in which only a person of the same type marries.) Then, if an M_{ML}-type woman meets a middle-type man, she marries him and learns that she is a middle-type.

[22] A belief-updating process of low-type women is as follows. An L_0-type woman becomes the 'L_{ML}-type' after she is rejected by a high-type man. An L_{ML}-type woman rejects a low-type man in the PSEI, similarly to an M_{ML}-type woman. Thus, an L_{ML}-type woman becomes the 'L_{L1}-type' by meeting with a middle-type man. Another L_0-type woman becomes the 'L_{L2}-type,' after she meets a middle-type man.

[23] As we show in the following, these proportions depend on $F_i(x)$ $(i = m, w)$. Therefore, we can rewrite these proportions using λ_k^i $(k = H, M, L)$.

confirm her actual type.[24] This is because, if the actual type of an l_{ML}-type woman is a low-type, she will marry a low-type man sooner or later regardless of her behavior. At this time, she obtains the same value when she is single regardless of her behavior due to a lack of time consuming cost ($r = 0$). [25] Hence, the possibility that the actual type of an l_{ML}-type woman is the low-type does not affect the decision of an l_{ML}-type woman. Consequently, the decision of an l_{ML}-type woman is the same as that of a middle-type woman with perfect self-knowledge.

If $r > 0$, the possibility that the actual type of an l_{ML}-type woman is a low-type affects her own decision. The agents with imperfect self-knowledge need to take into account the time-consuming cost due to the belief-updating process.[26] When an l_{ML}-type woman is the low-type, she is refused by a middle-type man. It is then desirable for an L_{ML}-type woman to accept a low-type man before thoroughly understanding her own type.[27] Therefore, the reservation utility of an l_{ML}-type ($l = M, L$) woman ($R^p_{l_{ML}}$) is lower than that in the case of $r = 0$.[28] A similar discussion could be presented for $R^p_{j_{HM}} = R^p_{k_0} = R^*_H (= x_H)$ when $r = 0$.

Lemma 2 means that an l_{ML}-type woman rejects (accepts) a low-type man if there are enough (few enough) middle-type men or if there are enough (few enough) M_{ML}-type women. Especially, the increase in the probability that the actual type of an l_{ML}-type woman is the middle-type ($\mu\lambda^w_M$) raises the reservation utility level of an l_{ML}-type woman. Similarly, a j_{HM}-type woman rejects (accepts) a middle-type man if there are enough (few enough) high-type men or if there are enough (few enough) H_{HM}-type women. In the following analysis, we assume that $R^p_{l_{ML}} > x_L$ and $R^p_{k_0} > x_M$ hold to focus on the PSEI.

Lemma 2 also shows that the reservation utility of a k_0-type woman for a middle-type man is the same as that of a j_{HM}-type woman. If the actual type of a k_0-type woman is the low-type, she will be rejected by both a high-type man and a middle-type man regardless of her own behavior. Therefore, the decision of a k_0-type woman whether or not to accept a middle-type man does not depend on the possibility that she is a low-type. Hence, even if a k_0-type woman becomes a j_{HM}-type woman by meeting a low-type man, her decision will not change.

Finally, we derive the sufficient conditions for the PSEI. In the PSEI, we can rewrite R^p_{Mm} in (4) as

$$R^p_{Mm} \equiv \frac{\alpha(\mu+\gamma\phi)\ \lambda^w_M x_M}{\left(r+\alpha(\mu+\gamma\phi)\ \lambda^w_M\right)} \tag{7}$$

by replacing η and ζ by $(\mu + \gamma\phi)$ and $(\psi + \kappa)$, respectively, from Figure 1.

Given the above matching strategies, $x_L < R^p_{Mm} (< R^*_M)$, $x_L < R^p_{l_{ML}} (< R^*_M)$, and $x_M <$

[24] An L_{ML}-type woman will be apparently overconfident as she raises her reservation utility to reject a low-type man. This is because that she accepts him when she know her own type. We will analyze the case of apparent overconfidence in Subsubsection 3.2.2.

[25] When $r = 0$, $V^r_{L_{ML}}$ in (35) equals $V^a_{L_{ML}}$ in (37) .

[26] Therefore, in our model, it is possible that a woman with imperfect self-knowledge could marry before thoroughly understanding her own type.

[27] When $r > 0$, $V^r_{L_{ML}}$ in (35) is always larger than $V^a_{L_{ML}}$ in (37).

[28] If a l_{ML}-type woman lowers her $R^p_{l_{ML}}$ to accept a low-type man whom she rejects when she knows her own type, a M_{ML}-type woman will be apparently underestimate her own type. We will analyze this case of apparent underconfidence in Subsubsection 3.2.3.

$R^p_{j_{HM}} = R^p_{k_0} (< R^*_H)$ hold in the PSEI. Moreover, the steady state requires

$$\begin{aligned}
\alpha\lambda^m_M (1-\theta)\lambda^w_H &= \alpha\lambda^m_H \theta\lambda^w_H, &(8)\\
\alpha\lambda^m_H (1-\mu-\gamma)\lambda^w_M &= \alpha\lambda^m_M \mu\lambda^w_M, &(9)\\
\alpha\lambda^m_M (1-\mu-\gamma)\lambda^w_M &= \alpha\lambda^m_H \gamma (1-\phi)\lambda^w_M = \alpha\lambda^m_M \gamma\phi\lambda^w_M, &(10)\\
\alpha\lambda^m_H (1-\psi-\kappa)\lambda^w_L &= \alpha\lambda^m_M \psi (1-\nu)\lambda^w_L = \alpha\lambda^m_L \psi\upsilon\lambda^w_L, &(11)\\
\alpha\lambda^m_M (1-\psi-\kappa)\lambda^w_L &= \alpha\lambda^m_L \kappa\lambda^w_L. &(12)
\end{aligned}$$

Equation (8) means that the rate at which H_0-type women learn that they are the high- or the middle-type (that is, H_0-type women become H_{HM}-type women) equals the rate at which H_{HM}-type women marry high-type men (see also Figure 1). Equation (9) means that the rate at which M_0-type women learn that they are the middle- or the low-type equals the rate at which M_{ML}-type women marry middle-type men. The first equality of (10) suggests that the rate at which M_0-type women become M_{HM}-type women equals the rate at which M_{HM}-type women become M_M-type women. The second equality of (10) means that the rate at which M_{HM}-type women become M_M-type women equals the rate at which M_M-type women marry middle-type men. The first equality of (11) means that the rate at which L_0-type women become L_{ML}-type women equals the rate at which L_{ML}-type women become L_{L1}-type women. The second equality of (11) means that the rate at which L_{ML}-type women become L_{L1}-type women equals the rate at which L_{L1}-type women marry low-type men. Equation (12) means that the rate at which L_0-type women become L_{L2}-type women equals the rate at which L_{L2}-type women marry low-type men and leave the market.

From (8) - (12), we then obtain the next proposition for the PSEI.

Proposition 2 *(PSEI) Let us assume that $x_M < R^*_H$ and $x_L < R^*_M$ hold. If*

$$\begin{aligned}
R^p_{Mm} &= \frac{\lambda^m_H \alpha\lambda^w_M x_M}{r(\lambda^m_H+\lambda^m_M)+\alpha\lambda^m_H\lambda^w_M} > x_L,\\
R^p_{l_{ML}} &= \frac{\alpha\lambda^m_H(\lambda^m_L+\lambda^m_M)\lambda^m_M\lambda^w_M x_M (r+\alpha\lambda^m_L)}{\lambda^w_M\lambda^m_H(\lambda^m_L+\lambda^m_M)(r+\alpha\lambda^m_M)(r+\alpha\lambda^m_L)+\lambda^w_L r\lambda^m_L(\lambda^m_H+\lambda^m_M)(r+\alpha\lambda^m_L+\alpha\lambda^m_M)} > x_L,\\
R^p_{j_{HM}} &= R^p_{k_0} = \frac{\alpha\lambda^m_H\lambda^w_H(\lambda^m_H+\lambda^m_M)x_H(r+\alpha\lambda^m_M)}{\lambda^w_H(\lambda^m_H+\lambda^m_M)(r+\alpha\lambda^m_M)(r+\alpha\lambda^m_H)+r\lambda^m_M\lambda^w_M(r+\alpha\lambda^m_H+\alpha\lambda^m_M)} > x_M,
\end{aligned}$$

there exists the PSE with imperfect self-knowledge (PSEI) in which high-type agents form the first cluster of marriages, middle-type men and M_{ML}- and M_M-type women, the second cluster, and low-type men and L_{L1}- and L_{L2}-type women, the third cluster.

Proof. See, Appendix A. ■

As Proposition 2 shows, in the steady-state equilibrium with imperfect self-knowledge, the reservation utilities of men (women) depend both on $F_m(x)$ and $F_w(x)$. On the other hand, when all agents know their own types, in the steady state, the reservation utilities of men (women) depend only on the type distribution of women (men) $F_i(x)$ $(i = m, w)$.

The implications of Proposition 2 are as follows: if there are enough high-type men or if there are enough high-type women ($R^p_{j_{HM}} = R^p_{k_0} > x_M$), a middle-type man would be rejected by a k_0-type and a j_{HM}-type ($j = H, M$) woman. However, a middle-type man rejects a low-type woman when there are enough M_M- and M_{ML}-type women ($R^p_{Mm} > x_L$) who accept middle-type men.[29] Furthermore, if there are enough middle-type men ($R^p_{l_{ML}} > x_L$), an l_{ML}-type ($l = M, L$) woman would reject a low-type man.[30] However, when an L_{ML}-type woman rejects a low-type man, she becomes an L_{L1}-type woman sooner or later due to being rejected by a middle-type man. Then, a low-type woman who learns that she is a low-type (namely, she is L_{L1}-type or L_{L2}-type) accepts a low-type man. As a result, the PSEI occurs. It is noteworthy that the first cluster of marriages is not influenced by women who are unaware of their own types.

3.2.2 Case of apparent overconfidence

In this subsubsection, we consider the *case of apparent overconfidence*: A woman with imperfect self-knowledge apparently overestimates her actual type if she rejects a man whom she accepts when she knows her own type. This apparent overconfidence is generated due to a correct belief-updating process, i.e., agents have no false information and follow Bayes's rule in updating their posterior beliefs about their own types. Thus, we use the term *apparent* overconfidence to distinguish it from *true* overconfidence, which is generally generated due to some errors in an agent's processing information.

We show that the apparent overconfidence of some women generates two externalities in this subsubsection. The apparent overconfidence of a woman delays the time until marriage of the man who is rejected by her relative to that in the benchmark case. Then, the apparent overconfidence has a *direct negative externality.* Furthermore, the apparent overconfidence of some women may have an *indirect externality:* when there are many women who are apparently overconfident in the market, the men who have now been rejected by these women change their behavior; i.e., they accept another type of women whom these men have previously rejected. Given this, in a two-sided search, the women who have now been accepted by these men may also change their behavior, i.e., they may reject the men whom they had accepted previously. Therefore, it is possible that the indirect externality of apparent overconfidence may spread to lower-type agents than to apparently overconfident agents. In this subsubsection, we show that the indirect externality of the apparent overconfidence prevents the lowest-type men from marrying in an equilibrium. For this, we will find a *Type 1 equilibrium (T1E)* in which high-type men reject middle-type women, middle- and low-type men accept low-type women, k_0-type ($k = H, M, L$) women reject middle-type men, and l_{ML}-type ($l = M, L$) women reject low-type men.[31] Then, M_0- and L_0-type women are apparently overconfident as they reject the men whom they would accept with knowledge of their own types.[32] Since apparently overconfident M_0-type women reject middle-type men,

[29]The proportion of M_M- and that of M_{ML}-type women in the market depend on λ^m_H. Then, if there are many high-type men, there will be many M_M- and M_{ML}-type women.

[30]If a l_{ML}-type ($l = M, L$) woman accepts a low-type man, men and women of different types would marry. Then, a PSEI does not occur.

[31]The following analysis shows that there are no j_{HM}-type women in the T1E.

[32]This is the simplest case in which apparent overconfidence occurs. Of course, we can consider other cases

middle-type men accept low-type women.

First, we investigate the optimal strategies of men when there are apparently overconfident women. Now, a high-type man has the same reservation utility as a high-type man in the PSE. The option of a middle-type man is to marry or to turn down a low-type woman, as a high- or M_0-type woman rejects him in the T1E. Then, in the same manner as in Lemma 1, the reservation utility of a middle-type man for a low-type woman $R_{Mm}^{T1} \equiv \frac{\alpha\eta\lambda_M^w x_M}{(r+\alpha\eta\lambda_M^w)}$ $(< R_M^*)$ is immediately obtained. Let us assume that $R_{Mm}^{T1} \leq x_L$ in the following analysis in order to explore the influence of the indirect externality of apparent overconfidence. A low-type man also accepts a low-type woman because a middle-type man accepts a low-type woman.

Next, we investigate the strategies of women given men's behavior: $R_H^* > x_M$ and $R_{Mm}^{T1} \leq x_L$. Now, since a middle- or a low-type man accepts any woman, the women who receives his proposal learns nothing about her own type. Then, only when a woman meets a high-type man, she learns something about her own type. Figure 2 describes the women's belief-updating processes.[33] Then, there are five kinds of women according to different beliefs: k_0-type women $(k = H, M, L)$ and l_{ML}-type women $(l = M, L)$. Here, $\tau \in (0,1)$ and $\varpi \in (0,1)$ denote the proportion of M_{ML}-type women and L_{ML}-type women, respectively. The optimal strategies of these women are obtained in the next lemma.

Lemma 3 *Let us assume that a high-type man rejects a middle-type woman* $(R_H^* > x_M)$, *a middle-type man accepts a low-type woman, and a low-type man accepts a low-type woman* $(R_{Mm}^{T1} \leq x_L < R_M^*)$. *Since*

$$x_L < R_{l_{ML}}^{T1} \equiv \frac{\alpha\lambda_M^m x_M}{r+\alpha\lambda_M^m} = R_M^*,$$

an l_{ML}*-type woman* $(l = M, L)$ *rejects a low-type man. On the other hand, a* k_0*-type woman rejects (accepts) a middle-type man if*

$$x_M < (\geq) \frac{\lambda_H^w\lambda_H^m\alpha x_H\left(r+\alpha\lambda_M^m\right)}{\left(\lambda_H^w\left(r+\alpha\lambda_M^m\right)\left(r+\alpha\lambda_H^m\right)+r(1-\tau)\left(\lambda_L^w+\lambda_M^w\right)\left(r+\alpha\lambda_H^m+\alpha\lambda_M^m\right)\right)} \equiv R_{k_0}^{T1}, \tag{13}$$

where $R_{k_0} < R_H^*$.

Proof. See, Appendix A. ■

Lemma 3 suggests that a k_0-type woman rejects a middle-type man when there are enough high-type men or if there are enough high-type women. On the other hand, since a middle-type man accepts a low-type woman, an M_{ML}-type woman and an L_{ML}-type woman face the same problem. They decide whether to accept or reject low-type men. As a result, an l_{ML}-type $(l = M, L)$ woman turns down a low-type man, as there are enough middle-type men $(x_L < R_M^*)$.

in order to describe the apparently overconfident behavior. The reason for this assumption will be discussed in Section 5.

[33] If a high-type woman is accepted by a high-type man, she leaves the market with him and knows that she is a high-type. After a middle- or a low-type woman is rejected by a high-type man, she becomes an l_{ML}-type woman $(l = M, L)$. However, she does not learn any more about her own type, since the offer from a middle- or a low-type man carries no information about the types of women.

Given matching strategies of men and women, let us derive conditions for the T1E in which $R_{Mm}^{T1} \le x_L$ and $R_{k_0}^{T1} > x_M$ hold. At this time, we can rewrite R_{Mm}^{T1} as

$$R_{Mm}^{T1} \equiv \frac{\alpha\tau\lambda_M^w x_M}{\left(r+\alpha\tau\lambda_M^w\right)} \left(< R_M^*\right) \tag{14}$$

by replacing η and ζ by τ and ϖ, respectively. Moreover, a steady state requires

$$\begin{aligned} \alpha\lambda_H^m (1-\tau)\lambda_M^w &= \alpha\lambda_M^m \tau\lambda_M^w, &(15)\\ \alpha\lambda_H^m (1-\varpi)\lambda_L^w &= \alpha\lambda_M^m \varpi\lambda_L^w. &(16)\end{aligned}$$

Equation (15) means that the rate at which M_0-type women become M_{ML}-type women equals the rate at which M_{ML}-type women marry middle-type men. Likewise, Equation (16) means that the rate at which L_0-type women become L_{ML}-type women equals the rate at which L_{ML}-type women marry middle-type men.

We then obtain the following proposition. In this equilibrium, the indirect externality of apparent overconfidence of M_0-type women prevents the lowest-type men from marrying.

Proposition 3 *(T1E) Let us assume that $x_M < R_H^*$ and $x_L < R_M^*$ hold. If*

$$\left(R_H^* >\right) R_{k_0}^{T1} = \frac{\lambda_H^w\lambda_H^m\alpha x_H\left(r+\alpha\lambda_M^m\right)\left(\lambda_H^m+\lambda_M^m\right)}{\lambda_H^w\left(r+\alpha\lambda_M^m\right)\left(r+\alpha\lambda_H^m\right)\left(\lambda_H^m+\lambda_M^m\right)+r\lambda_M^m\left(\lambda_L^w+\lambda_M^w\right)\left(r+\alpha\lambda_H^m+\alpha\lambda_M^m\right)} > x_M,$$

and

$$R_{Mm}^{T1} = \frac{\alpha\lambda_H^m\lambda_M^w x_M}{r\left(\lambda_H^m+\lambda_M^m\right)+\alpha\lambda_H^m\lambda_M^w} \le x_L,$$

there exists the T1E in which high-type agents form the first cluster of marriages, middle-type men and M_{ML}-type women, the second cluster, and middle-type men and L_{ML}-type women, the third cluster. In this equilibrium, low-type men can never marry.

Proof. See, Appendix A. ■

The implications of Proposition 3 are as follows: when there are enough high-type women ($R_{k_0}^{T1} > x_M$), a k_0-type woman rejects a middle-type man. M_0- and L_0-type women become M_{ML}- and L_{ML}-type women, respectively, after they meet high-type men. If there are few enough M_{ML}-type women ($R_{Mm}^{T1} \le x_L$) who accept middle-type men, a middle-type man accepts a low-type woman. This leads an M_{ML}- or an L_{ML}-type woman to refuse the offer by a low-type man, since there are enough middle-type men who accept her ($R_{l_{ML}} = R_M^* > x_L$). As a result, a low-type man can never marry.[34] The first cluster of marriages is not influenced by women who do not know their own types. It is noteworthy that middle- and low-type women marry middle-type men before they thoroughly know their actual type.

In the next subsubsection, we investigate a case in which some middle-type women do not know their own types and accept low-type men, i.e., a case of apparent underconfidence.

[34] The proportion of l_{ML}-type women ($l = M, L$) depends on the proportion of high-type men λ_H^m. However, even if the actual type distribution of men and that of women are symmetric, $R_{Mm}^{T1} \le x_L$ and $R_{k_0}^{T1} > x_M$ hold in some parameter ranges.

3.2.3 Case of apparent underconfidence

In this subsubsection, let us consider the *case of apparent underconfidence*: a woman apparently underestimates her own type when she accepts a man whom, when she knows her own type, she rejects.[35]

The woman's apparent underconfidence makes the future partner better off, as she increases the value of the match to the partner. Then, the apparent underconfidence will have a direct positive externality. However, as we show in this subsubsection, there is no indirect externality in the case of apparent underconfidence. To see this, we start by considering a *Type 2 equilibrium candidate (T2EC)* in which high-type men reject middle-type women, middle- and low-type men reject low-type women, k_0- and j_{HM}-type ($k = H, M, L,\ j = H, M$) women reject middle-type men, and l_{ML}-type ($l = M, L$) women accept low-type men.[36] That is, high-type agents form the first cluster of marriages, middle-type men and M_{ML}- type and M_M-type women, the second cluster, low-type men and M_{ML}-type women, the third cluster, and low-type women cannot marry due to the rejection from low-type men. Consequently, the apparent underconfidence of M_{ML}-type women prevents the lowest-type women from marrying in the T2EC.[37] The assumption that k_0- and j_{HM}-type women reject middle-type men means that M_0-, L_0-, and M_{HM}-type women are apparently overconfident. However, to remove the indirect externality of this apparent overconfidence, we now assume that middle-type men do not change their behavior compared to that in the benchmark case.

First, we investigate the strategies of men given the above women's behavior. A high-type man has the same reservation utility as a high-type man in the benchmark case. In the same manner as in Lemma 1, the reservation utility of a middle-type man for a low-type woman $R^{T2}_{Mm} = \frac{\alpha\eta\lambda^w_M x_M}{(r+\alpha\eta\lambda^w_M)}$ $(< R^*_M)$ is obtained. The option of a low-type man is to marry or to turn down a low-type woman, as some apparently underconfident middle-type women accept low-type men. Let us assume that $\varphi \in (0, 1)$ of middle-type women accept low-type men due to their apparent underconfidence. Thus, the reservation utility of a low-type man for a low-type woman is obtained in the same manner as that for a middle-type man. His reservation utility is $R^{T2}_{Lm} \equiv \frac{\alpha\varphi\lambda^w_M x_M}{r+\alpha\varphi\lambda^w_M}$, where $R^{T2}_{Lm} < R^*_M$. We assume that $R^{T2}_{Mm} > x_L$ and $R^{T2}_{Lm} > x_L$ in the following analysis in order to consider the T2EC.

Next, we investigate the strategies of women given $x_L < R^{T2}_{Mm}$ and $x_L < R^{T2}_{Lm}$. At this time, the means of the proposal and rejection by a high-type man are the same as those in the PSEI. Rejection of a woman by a middle- or low-type man means that the woman is the low-type. An offer from a middle- or a low-type man means that the woman who is accepted by him is either the high- or middle-type.

Given $x_L < R^{T2}_{Mm}$ and $x_L < R^{T2}_{Lm}$, women's belief- updating processes are described in

[35] From Lemma 2, if $r > 0$ is large, a woman with imperfect knowledge would tend to underestimate her own type.

[36] In the following analysis, it is shown that there actually exist j_{HM}-type and l_{ML}-type women in the T2EC.

[37] Adoption of the assumption that M_{ML}-type women are apparently underconfident is to reuse the framework of the PSEI of Subsection 3.2.1 in the latter half of this subsubsection. Of course, another case of apparent underconfidence can be considered, which we discuss in the later Section 5.

Figure 3.[38] Then, there are eight kinds of women according to different beliefs in the market: k_0-type ($k = H, M, L$), j_{HM}-type ($j = H, M$), l_{ML}-type ($l = M, L$), and M_M-type. Here, θ_2, μ_2, $\gamma_2(1-\phi_2)$, $\gamma_2\phi_2$, and $\psi_2 \in (0,1)$ denote the proportion of H_{HM}-type women, M_{ML}-type women, M_{HM}-type women, M_M-type women, and L_{ML}-type women, respectively.

The next Lemma implies that the indirect externality of the apparent underconfidence of M_{ML}-type women does not occur.

Lemma 4 *Let us assume that $x_M < R_H^*$, $x_L < R_M^*$. The T2EC in which $R_{Mm}^{T2} > x_L$, $R_{Lm}^{T2} > x_L \geq R_{l_{ML}}^{T2}$, $R_{j_{HM}}^{T2} > x_M$, and $R_{k_0}^{T2} > x_M$ ($l = M,L$, $j = H,M$, $k = H,M,L$) is not an equilibrium.*

Proof. See, Appendix A. ■

This Lemma implies that apparent underconfidence does not have indirect externality.[39] If a low-type man rejects a low-type woman by the expectation to marry an M_{ML}-type woman who is apparently underconfident, the proposal of a low-type man for an M_{ML}-type woman informs her that she is the middle-type. Furthermore, if the actual type of an l_{ML}-type woman is the low-type, she will be rejected by a middle- or a low-type man regardless of her behavior. Then, the possibility that an l_{ML}-type woman is the low-type does not affect her decision to accept a low-type man or not. As a result, an l_{ML}-type woman has the incentive to reject a low-type man, since she prefers to have the chance to learn her actual type than to accept a low-type man.[40] Therefore, a low-type man always accepts a low-type woman even if there are many M_{ML}-type women who accept low-type men.[41]

Finally, we show that the apparent underconfidence has direct externality in a steady state. Let us consider *a Type 3 equilibrium (T3E)* in which the behavior of agents, except low-type men, is same as that in the T2EC: any low-type man accepts a low-type woman. This acceptance of a low-type man for a low-type woman gives him a chance to marry an M_{ML}-type woman.

The optimal strategies of a middle- and a low-type man are obtained in the same manner as the T2EC. Let $R_{Mm}^{T3} \equiv \frac{\alpha\eta\lambda_M^w x_M}{(r+\alpha\eta\lambda_M^w)} (< R_M^*)$ and $R_{Lm}^{T3} \equiv \frac{\alpha\varphi\lambda_M^w x_M}{(r+\alpha\varphi\lambda_M^w)} (< R_M^*)$ denote the optimal strategy of middle-type and low-type men, respectively. Let us assume that $R_{Mm}^{T3} > x_L \geq$

[38]An H_0-type woman learns that she is the high-type when she meets a high-type man and marries him. Another H_0-type woman becomes an H_{HM}-type woman by meeting a middle- or a low-type man. A H_{HM}-type woman leaves the market when she meets a high-type man. An M_0-type woman becomes an M_{ML}-type woman by meeting a high-type man. Another M_0-type woman becomes an M_{HM}-type woman after meeting a middle- or a low-type man. An M_{HM}-type woman becomes an M_M-type woman after meeting a high-type man. An M_M-type woman leaves the market with a middle-type man when they meet. An L_0-type woman becomes a L_{ML}-type woman after meeting a high-type man. Another L_0- or an L_{ML}-type woman learns that she is the low-type after meeting a middle- or low-type man. Then, she leaves the market alone.

[39]In this subsubsection, we assume that M_{ML}-type women are apparently underconfident. The same result holds qualitatively in other cases of apparent underconfidence (we investigate the case in which H_0-type women are apparently underconfident in the Appendix B).

[40]This is because there are enough middle-type men in the market ($R_M^* > x_L$).

[41]However, if there are many M_{ML}-type women in the market, this large proportion of M_{ML}-type women in the market increases the possibility that an l_{ML}-type woman is the middle-type. Then, a l_{ML}-type woman raises her reservation utility for a low-type man. Therefore, if there are enough M_{ML}-type women in the market, an l_{ML}-type ($l = M, L$) woman rejects a low-type man. When an l_{ML}-type woman rejects a low-type man, the steady-state equilibrium will become the PSEI.

R_{Lm}^{T3} in the following analysis.[42] Since the behavior of men is the same as that in the PSEI, the information about women's types from the proposals or rejections by men is also the same as that in the PSEI. Then, the Bayesian updating processes of women are also the same as those in the PSEI (see Figure 4). Let $\theta_3 \in (0,1)$, $\gamma_3(1-\phi_3) \in (0,1)$, $\gamma_3\phi_3 \in (0,1)$, $\mu_3 \in (0,1)$, $\psi_3(1-\nu_3)$, $\psi_3\nu_3$ and $\kappa_3 \in (0,1)$ denote the proportion of H_{HM}-type women, that of M_{HM}-type women, that of M_M-type women, that of M_{ML}-type women, that of L_{ML}-type women, that of L_{L1}-type women, and that of L_{L2}-type women, respectively.[43]

Hence, the optimal strategies of an l_{ML}-type, a j_{HM}-type, and a k_0-type woman are obtained in the next lemma.

Lemma 5 *Let us assume that a high-type man rejects a middle-type woman ($R_H^* > x_M$), a middle-type man rejects a low-type woman $\left(R_M^* > R_{Mm}^{T3} > x_L\right)$, and a low-type man accepts a low-type woman $\left(R_{Lm}^{T3} \le x_L\right)$. If*

$$x_L < (\ge)\, R_{l_{ML}}^{T3} \equiv \frac{\lambda_M^w \lambda_M^m \alpha\mu_2 x_M\left(r+\alpha\lambda_L^m\right)}{r\psi_2(1-\nu_2)\lambda_L^w\left(r+\alpha\lambda_L^m+\alpha\lambda_M^m\right)+\mu_2\lambda_M^w\left(r+\alpha\lambda_L^m\right)\left(r+\alpha\lambda_M^m\right)}, \tag{17}$$

an l_{ML}-type women ($l = M, L$) rejects (accepts) a low-type man, where $R_{l_{ML}}^{T3} < R_M^*$. If

$$x_M < (\ge)\, R_{j_{HM}}^{T3} \equiv \frac{\alpha\theta_2\lambda_H^w\lambda_H^m x_H\left(r+\alpha\lambda_M^m\right)}{\left(r\lambda_M^w\gamma_2(1-\phi_2)\left(r+\alpha\lambda_H^m+\alpha\lambda_M^m\right)+\theta_2\lambda_H^w\left(r+\alpha\lambda_H^m\right)\left(r+\alpha\lambda_M^m\right)\right)} = R_{k_0}^{T3}, \tag{18}$$

a j_{HM}-type ($j = H, M$) and a k_0-type woman reject (accept) a middle-type man, where $R_{j_{HM}}^{T3} = R_{k_0}^{T3} < R_M^*$.

Proof. See, Appendix A. ■

Lemma 5 implies as follows: Equations (17) and (18) are the same form as Equations (5) and (6). Then, in contrast with the PSE, the reservation utility level of an l_{ML}-type ($l = M, L$), that of a k_0-type ($k = H, M, L$), and that of a j_{HM}-type ($j = H, M$) woman are lowered or raised for the same reason as in Lemma 2.

In the T3E, a k_0- or a j_{HM}-type woman rejects a middle-type man ($x_M < R_{j_{HM}}^{T3} = R_{k_0}^{T3}$), and an l_{ML}-type woman accepts a low-type man ($x_L \ge R_{l_{ML}}^{T3}$). That is, an M_{ML}-type woman is apparently underconfident. Given these women's behavior, we can rewrite R_{Mm}^{T3} and R_{Lm}^{T3} as

$$R_{Mm}^{T3} = \frac{\alpha(\mu_3+\gamma_3\phi_3)\lambda_M^w x_M}{\left(r+\alpha(\mu_3+\gamma_3\phi_3)\lambda_M^w\right)}\,(> x_L)\,, \tag{19}$$

$$R_{Lm}^{T3} = \frac{\alpha\mu_3\lambda_M^w x_M}{r+\alpha\mu_3\lambda_M^w}\,(\le x_L) \tag{20}$$

by replacing μ and φ by $(\mu_3 + \gamma_3\phi_3)$ and μ_3, respectively (see Figure 4). When $x_M < R_{j_{HM}}^{T3}$,

[42] At this time, $\varphi < \eta$ is required. However, the following analysis shows that $\varphi < \eta$ holds.

[43] The difference between Figure 1 and Figure 3 is whether l_{ML}-type women accept low-type men or not. Therefore, $G_w(x)$ in the T3E is also different from that in the PSEI, although there are the same kinds of women according to their belief differences as those in the PSEI.

$x_L \geq R^{T3}_{l_{ML}}$ and $R^{T3}_{Mm} > x_L \geq R^{T3}_{Lm}$, the steady state requires

$$\begin{aligned}
\alpha\lambda_M^m (1-\theta_3)\lambda_H^w &= \alpha\lambda_H^m\theta_3\lambda_H^w, & (21)\\
\alpha\lambda_H^m (1-\mu_3-\gamma_3)\lambda_M^w &= \alpha(\lambda_M^m+\lambda_L^m)\mu_3\lambda_M^w, & (22)\\
\alpha\lambda_M^m (1-\mu_3-\gamma_3)\lambda_M^w &= \alpha\lambda_H^m\gamma_3(1-\phi_3)\lambda_M^w = \alpha\lambda_M^m\mu_3\phi_3\lambda_M^w, & (23)\\
\alpha\lambda_H^m (1-\psi_3-\kappa_3)\lambda_L^w &= \alpha(\lambda_M^m+\lambda_L^m)\psi_3(1-\nu_3)\lambda_L^w & (24)\\
\alpha\lambda_M^m\psi_3(1-\nu_3)\lambda_L^w &= \alpha\lambda_L^m\psi_3\nu_3\lambda_L^w, & (25)\\
\alpha\lambda_M^m (1-\psi_3-\kappa_3)\lambda_L^w &= \alpha\lambda_L^m\kappa_3\lambda_L^w. & (26)
\end{aligned}$$

All equations, except (22) and (24), have the same forms as (8) and (10)-(12) in the PSEI. Equation (22) means that the rate at which M_0-type women become M_{ML}-type women equals the rate at which M_{ML}-type women marry middle- or low-type men. Equation (24) means that the rate at which L_0-type women become L_{ML}-type women equals the rate at which L_{ML}-type women become L_{L1}-type women and the rate at which L_{ML}-type women marry low-type men.

From (21) - (26), we have the next proposition for the T3E. In this equilibrium, apparent underconfidence has direct externality.

Proposition 4 *(T3E) Let us assume that*

$$\begin{aligned}
R^{T3}_{Lm} &= \frac{\alpha(\lambda_H^m)^2\lambda_M^w x_M}{\lambda_H^m(r+\alpha\lambda_H^m\lambda_M^w)+r(\lambda_H^m+\lambda_M^m)(\lambda_L^m+\lambda_M^m)} \leq x_L\\
&< R^{T3}_{Mm} = \frac{\alpha\lambda_M^w x_M\lambda_H^m}{r(\lambda_H^m+\lambda_M^m)(\lambda_L^m+\lambda_M^m)+\lambda_H^m(r+\alpha\lambda_M^w)}\left(< R_M^*\right),\\
x_M &< R^{T3}_{j_{HM}} = R^{T3}_{k_0}\\
&= \frac{\alpha\lambda_H^m\lambda_H^w x_H(r+\alpha\lambda_M^m)(\lambda_H^m+(\lambda_H^m+\lambda_M^m)(\lambda_L^m+\lambda_M^m))}{\lambda_H^w(r+\alpha\lambda_M^m)(r+\alpha\lambda_H^m)(\lambda_H^m+(\lambda_H^m+\lambda_M^m)(\lambda_L^m+\lambda_M^m))+r\lambda_M^w(\lambda_H^m+\lambda_M^m)(\lambda_L^m+\lambda_M^m)(r+\alpha\lambda_H^m+\alpha\lambda_M^m)}\left(< R_H^*\right),\\
x_L &\geq R^{T3}_{l_{ML}} = \frac{\alpha\lambda_H^m\lambda_M^m\lambda_M^w x_M(\lambda_L^m+\lambda_M^m)(r+\alpha\lambda_L^m)}{\lambda_L^w r\lambda_L^m(r+\alpha\lambda_L^m+\alpha\lambda_M^m)(\lambda_H^m+(\lambda_H^m+\lambda_M^m)(\lambda_L^m+\lambda_M^m))+\lambda_M^w\lambda_H^m(\lambda_L^m+\lambda_M^m)(r+\alpha\lambda_M^m)(r+\alpha\lambda_L^m)}.
\end{aligned}$$

There exists the T3E in which high-type agents form the first cluster of marriages, middle-type men and M_{ML}-type and M_M-type women, the second cluster, low-type men and M_{ML}-type women, the third cluster, and low-type men and L_{ML}-type, L_{L1}-type, and L_{L2}-type women, the fourth cluster.

Proof. See, Appendix A. ■

The implications of Proposition 4 are as follows: in the T3E, there is the direct externality of apparent underconfidence. Since there is no indirect externality of apparent underconfidence in this equilibrium, all agents can marry. If there are enough high-type men and women ($x_M < R^{T3}_{j_{HM}} = R^{T3}_{k_0} < R_H^*$), a middle-type man is rejected by a k_0-type and a j_{HM}-type woman. Moreover, a large proportion of high-type men implies a large proportion of M_M- and M_{ML}-type women in the market. As a result, if there are enough M_M- and M_{ML}-type women ($x_L < R^{T3}_{Mm}$), a middle-type man rejects a low-type woman. Furthermore, if there are few enough M_{ML}-type women satisfying $R^{T3}_{l_{ML}} \leq x_L\ (< R^{T3}_{Mm})$, an l_{ML}-type woman accepts a low-type man. This is because she assigns low probability to being a middle-type woman. However, a low-type man accepts a low-type woman, as there are few enough M_{ML}-type women ($R^{T3}_{Lm} \leq x_L$). Therefore, all agents can marry sooner or later.

From Propositions 2, 3 and 4, in some parameter ranges, multiple equilibria can occur: both the PSEI and the T3E exist. To clarify this, we consider the next example.

Example 1 *Let us assume that* $2\alpha > 3r$ *and that* $F_m(x)$ *and* $F_w(x)$ *are discrete uniform distributions:* $\lambda^i_k = \frac{1}{3}$, $(i = m, w,\ k = H, M, L)$*. At this time, the sufficient conditions for the PSEI become* $R^p_{j_{HM}} = R^p_{k_0} = \frac{(3r+\alpha)2\alpha x_H}{18r\alpha+2\alpha^2+27r^2} > x_M$*,* $R^p_{Mm} = \frac{\alpha x_M}{6r+\alpha} > R^p_{l_{ML}} = \frac{(3r+\alpha)\alpha x_M}{12r\alpha+\alpha^2+18r^2} > x_L$ *from Proposition 2. Similarly, the sufficient conditions for the T1E are* $R^{T1}_{k_0} = \frac{(3r+\alpha)\alpha x_H}{12r\alpha+\alpha^2+18r^2} > x_M$ *and* $R^{T1}_{Mm} = \frac{\alpha x_M}{6r+\alpha} \leq x_L$ *from Proposition 3. The sufficient conditions for the T3E are* $x_M < R^{T3}_{j_{HM}} = R^{T3}_{k_0} = \frac{(3r+\alpha)7\alpha x_H}{66r\alpha+7\alpha^2+99r^2}$*,* $R^{T3}_{Lm} = \frac{\alpha x_M}{21r+\alpha} < R^{T3}_{l_{ML}} = \frac{(3r+\alpha)2\alpha x_M}{26r\alpha+2\alpha^2+39r^2} \leq x_L < R^{T3}_{Mm} = \frac{\alpha x_M}{7r+\alpha}$ *from Proposition 4. Since* $R^{T3}_{Mm} - R^{T1}_{Mm} = -\frac{r\alpha x_M}{(7r+\alpha)(6r+\alpha)} < 0$*, the T3E and the T1E do not hold. Moreover, as* $R^{T1}_{Mm} = R^p_{Mm}$*, the PSEI and the T1E do not hold either. However, since* $x_M < R^{T3}_{k_0} < R^p_{k_0}$ *and* $R^{T3}_{l_{ML}} < x_L < R^p_{l_{ML}} < R^{T3}_{Mm}$*, the PSEI and the T3E hold.*

The intuition of multiple equilibria is as follows: marriage patterns are determined by the expectation of all agents about the behavior of agents with imperfect self-knowledge. If all agents expect that l_{ML}-type $(l = M, L)$ women will accept low-type men, these expectations form their prior beliefs $G_w(x)$. Then, the marriage pattern of the T3E occurs. On the other hand, if all agents expect that l_{ML}-type women will reject low-type men, the marriage pattern of the PSEI occurs through their prior belief $G_w(x)$. When all agents know their own types under the cloning assumption and the assumption of non-transferable utility, a unique equilibrium will occur (see Burdett and Coles (1997)). However, if there are agents with imperfect self-knowledge under the cloning assumption and the non-transferable utility, there are possibilities that multiple equilibria will occur.

The welfare implication of these two steady states is that the PSEI and the T3E are not Pareto-rankable: apparent underconfident women prefer the PSEI, while low-type men prefer the T3E. However, as shown in detail in Example 2 of Section 4, the welfare of marriages of the T3E is raised relative to that of the PSEI as a whole. This is because apparently underconfident women accept middle- and low-type men in the T3E and the number of marriages of the T3E is raised relative to that of the PSEI.

Propositions 3, Proposition 4 and Lemma 4 suggest that, whereas apparent overconfidence has an indirect externality, apparent underconfidence does not have indirect externality. This difference depends on the agents whom the indirect externality firstly affects. In the case of apparent underconfidence, some middle-type women with imperfect self-knowledge accept low-type men. Given this, if a low-type man rejects a low-type woman, his offer for an apparently underconfident middle-type woman informs her that she is worse off. Then, the indirect externality of apparent underconfidence does not always occur in equilibrium. On the other hand, in a case of apparent overconfidence, even if a middle-type man accepts a low-type woman due to the existence of many apparently overconfident middle-type women, the acceptance of a middle-type man for a low-type woman makes a low-type woman better off. Therefore, the indirect externality of apparent overconfidence remains.[44]

In the next section, we investigate the total number of marriages and the welfare of the economy.

4 Welfare and the number of marriages

It is meaningful to investigate whether the existence of women with the imperfect self-knowledge may improve the welfare of the economy relative to the benchmark case. In this section, we examine the total number of marriages and the welfare from new marriages that take place in the marriage market at any point in time.

First, we investigate the number of marriages at the PSE as a benchmark. In the PSE, a high-type man meets a high-type woman with probability $\alpha\lambda_H^w$, and there are $\lambda_H^m N$ number of high-type men in the market. Then, the number of marriages among high-type agents in a market is $\alpha\lambda_H^w\lambda_H^m N$. In the same way, we obtain the number of marriages of middle-type $\alpha\lambda_M^m\lambda_M^w N$ and low-type $\alpha\lambda_L^w\lambda_L^m N$. Therefore, the total number of marriages in the marriage market T^* is

$$T^* = \alpha\lambda_H^m\lambda_H^w N + \alpha\lambda_M^m\lambda_M^w N + \alpha\lambda_L^m\lambda_L^w N. \tag{27}$$

The number of marriages in the PSEI (T^p), in the T1E $\left(T^{T1}\right)$, and in the T3E $\left(T^{T3}\right)$ can be derived similarly (see also Figure 1-2,4). Therefore, we obtain

$$\begin{aligned}
T^p &= \alpha\lambda_H^m\lambda_H^w N + \alpha\left(\mu+\gamma\phi\right)\lambda_M^w\lambda_M^m N + \alpha\left(\psi v+\kappa\right)\lambda_L^w\lambda_L^m N, \\
&= \alpha\lambda_H^m\lambda_H^w N + \alpha\left(\tfrac{\lambda_H^m}{\lambda_H^m+\lambda_M^m}\right)\lambda_M^w\lambda_M^m N + \alpha\left(\tfrac{\lambda_M^m}{\lambda_L^m+\lambda_M^m}\right)\lambda_L^w\lambda_L^m N, && (28) \\
T^{T1} &= \alpha\lambda_H^m\lambda_H^w N + \alpha\tau\lambda_M^m\lambda_M^w N + \alpha\varpi\lambda_M^m\lambda_L^w N, \\
&= \alpha\lambda_H^m\lambda_H^w N + \alpha\left(\tfrac{\lambda_H^m}{\lambda_H^m+\lambda_M^m}\right)\lambda_M^m\lambda_M^w N + \alpha\left(\tfrac{\lambda_H^m}{\lambda_H^m+\lambda_M^m}\right)\lambda_M^m\lambda_L^w N && (29) \\
T^{T3} &= \alpha\lambda_H^m\lambda_H^w N + \alpha\left(\mu_2+\gamma_2\phi_2\right)\lambda_M^m\lambda_M^w N \\
&\quad +\alpha\mu_2\lambda_M^w\lambda_L^m N + \alpha\left(\psi_2+\kappa_2\right)\lambda_L^m\lambda_L^w N, \\
&= \alpha\lambda_H^m\lambda_H^w N + \alpha\left(\tfrac{\lambda_H^m}{\lambda_H^m+\left(\lambda_H^m+\lambda_M^m\right)\left(\lambda_L^m+\lambda_M^m\right)}\right)\lambda_M^m\lambda_M^w N \\
&\quad +\alpha\left(\tfrac{\left(\lambda_H^m\right)^2}{\lambda_H^m+\left(\lambda_M^m+\lambda_L^m\right)\left(\lambda_H^m+\lambda_M^m\right)}\right)\lambda_M^w\lambda_L^m N + \alpha\left(\lambda_H^m+\lambda_M^m\right)\lambda_L^m\lambda_L^w N. && (30)
\end{aligned}$$

Next, we explore welfare. If a high-type man marries a high-type woman, each of them obtains the utility of marriage x_H. Hence, in the PSE, the aggregation of high-type agents' utilities from marriage is $2\alpha\lambda_H^m\lambda_H^w x_H N$. Similarly, we obtain $2\alpha\lambda_M^m\lambda_M^w x_M N$ for the middle type and $2\alpha\lambda_L^m\lambda_L^w x_L N$ for the low type. As a result, the welfare of the whole society in the

[44] In the T1E, since middle-type men accept the lowest type of women (that is, low-type women), his offer for a low-type woman has no information about types of women. Then, a low- or a middle-type woman learns nothing from his offer. However, if there is a lower type than 'low-type' and if a middle-type man accpets a 'low-type' woman, a low- or a middle-type woman will learn something about her own type from the acceptance of a middle-type man. However, this acceptance also makes a low-type woman better off. Then, the indirect externality of apparent overconfidence will remain in this case.

PSE W^* is

$$W^* = \alpha\lambda_H^m\lambda_H^w (2x_H) N + \alpha\lambda_M^m\lambda_M^w (2x_M) N + \alpha\lambda_L^m\lambda_L^w (2x_L) N. \tag{31}$$

The welfare in the PSEI (W^p), in the T1E $\left(W^{T1}\right)$, and in the T3E $\left(W^{T3}\right)$ can be derived similarly. Hence,

$$W^p = \alpha\lambda_H^m\lambda_H^w (2x_H) N + \alpha\left(\frac{\lambda_H^m}{\lambda_H^m+\lambda_M^m}\right)\lambda_M^w\lambda_M^m (2x_M) N + \alpha\left(\frac{\lambda_M^m}{\lambda_L^m+\lambda_M^m}\right)\lambda_L^w\lambda_L^m (2x_L) N, \tag{32}$$

$$\begin{aligned} W^{T1} &= \alpha\lambda_H^m\lambda_H^w (2x_H) N \\ &+\alpha\left(\frac{\lambda_H^m}{\lambda_H^m+\lambda_M^m}\right)\lambda_M^m\lambda_M^w (2x_M) N + \alpha\left(\frac{\lambda_H^m}{\lambda_H^m+\lambda_M^m}\right)\lambda_M^m\lambda_L^w (x_M + x_L) N, \end{aligned} \tag{33}$$

$$\begin{aligned} W^{T3} &= \alpha\lambda_H^m\lambda_H^w (2x_H) N + \alpha\left(\frac{\lambda_H^m}{\lambda_H^m+(\lambda_H^m+\lambda_M^m)(\lambda_L^m+\lambda_M^m)}\right)\lambda_M^m\lambda_M^w (2x_M) N \\ &+\alpha\left(\frac{(\lambda_H^m)^2}{\lambda_H^m+(\lambda_M^m+\lambda_L^m)(\lambda_H^m+\lambda_M^m)}\right)\lambda_M^w\lambda_L^m (x_M + x_L) N + \alpha(\lambda_H^m+\lambda_M^m)\lambda_L^m\lambda_L^w (2x_L) N, \end{aligned} \tag{34}$$

hold. From these, the next lemma is immediately obtained.

Proposition 5 *The number of marriages and the welfare in the PSE are higher than those in the PSEI, i.e.,*

$$\begin{aligned} T^* &> T^p, \\ W^* &> W^p. \end{aligned}$$

Proof. From (27) and (28),

$$T^* - T^p = N\alpha\left((1-(\mu+\gamma\phi))\lambda_M^m\lambda_M^w + (1-(\kappa+v\psi))\lambda_L^m\lambda_L^w\right) > 0$$

holds. From (31) and (32), we also have

$$W^p - W^* = -\lambda_L^m\lambda_L^w x_L (1-(\kappa+v\psi)) - \lambda_M^m\lambda_M^w x_M (1-(\mu+\gamma\phi)) < 0.$$

■

From this proposition, the imperfect self-knowledge always lowers the number of marriages and the welfare. This is because the marriages of all agents, except high-type men and some high-type women who meet high-type men in their first encounter, are delayed by the refusal by the Bayesian updating process of women. The duration until marriage of each agent can be obtained easily. In the PSE, the duration until marriage of k-type man (woman) is $\frac{1}{\alpha\lambda_k^i}(i = m, w,\ k = H, M, L)$. However, in the PSEI, the duration until marriage of a middle-type man is $\frac{1}{\alpha\lambda_M^w}\frac{\lambda_H^m+\lambda_M^m}{\lambda_H^m}$, that of a low-type man is $\frac{1}{\alpha\lambda_L^w}\frac{\lambda_L^m+\lambda_M^m}{\lambda_M^m}$, that of an H_{HM}-type woman is $\frac{1}{\alpha\lambda_H^m\alpha\lambda_M^m}$, that of an M_{ML}-type woman is $\frac{1}{\alpha\lambda_H^m\alpha\lambda_M^m}$, that of an M_M-type woman is $\frac{1}{\alpha\lambda_H^m\alpha\lambda_M^m\alpha\lambda_L^m}$, that of an L_{L1}-type woman is $\frac{1}{\alpha\lambda_H^m\alpha\lambda_M^m\alpha\lambda_L^m}$, and that of L_{L2}-type woman is $\frac{1}{\alpha\lambda_M^m\alpha\lambda_L^m}$. Therefore, their marriages are delayed by the belief-updating process of women. However, the duration until marriage of an H_0-type woman who meets a high-type man in her first encounter and that of a high-type man are not influenced by the belief-updating process of women.

The number of marriages and the welfare in the T1E increase or decrease relative to the PSE by the following factors: the number of marriages and the welfare of middle-type women are always worse than those in the PSE, since their marriages are delayed due to their belief-updating process. Moreover, the number of marriages and the welfare of low-type men in a Type 1 economy are worse, as these men cannot get married. However, middle-type men accept both middle-type and low-type women in the T1E. Then, if $\lambda_H^m \lambda_L^w > (\leq) \lambda_M^m \lambda_M^w$, their number of marriages increases (decreases) as a whole. Moreover, if $\lambda_H^m \lambda_L^w x_L > (\leq) \lambda_M^m \lambda_M^w x_M$, the welfare of middle-type men increases (decreases). Now, low-type women can marry middle-type men. Therefore, their number of marriages increases (decreases) if $(\lambda_H^m + \lambda_M^m) \lambda_L^m < (\geq) \lambda_H^m \lambda_M^m$, and their welfare increases (decreases) if $(\lambda_H^m + \lambda_M^m) \lambda_L^m x_L < (\geq) \lambda_H^m \lambda_M^m x_M$.

On the other hand, the number of marriages and the welfare in the T3E increase or decrease relative to those in the PSE by the following factors: The number of marriages and the welfare of low-type women are always lowered, since these women learn their own types. The number of marriages and welfare of middle-type men also decrease, as these men are rejected by some middle-type women with imperfect self-knowledge. The middle-type women who marry low-type men obtain lower utilities than middle-type women with perfect self-knowledge. However, middle-type women may marry earlier than the PSE, since they accept middle- and low-type men. Then, if $\lambda_H^{m2} \lambda_L^m < (\geq) \lambda_M^m (\lambda_H^m + \lambda_M^m)(\lambda_L^m + \lambda_M^m)$, their number of marriages is lower (higher) than that in the PSE. Furthermore, their welfare is lower (higher) than that in the PSE if $\lambda_H^{m2} \lambda_L^m x_L < (\geq) \lambda_M^m x_M (\lambda_H^m + \lambda_M^m)(\lambda_L^m + \lambda_M^m)$. Now, low-type men can marry middle- and low-type women in the T3E. Therefore, the number of marriages to low-type men is raised (lowered) if $\lambda_H^{m2} \lambda_M^w > (\leq) \lambda_L^w \lambda_L^m (\lambda_H^m + (\lambda_H^m + \lambda_M^m)(\lambda_L^m + \lambda_M^m))$, and the welfare of low-type men is raised (lowered) if $\lambda_H^{m2} \lambda_M^w x_M > (\leq) \lambda_L^w x_L \lambda_L^m (\lambda_H^m + (\lambda_H^m + \lambda_M^m)(\lambda_L^m + \lambda_M^m))$.

In both the T1E and the T3E, the number of marriages and the welfare of high-type men/women are not influenced by the imperfect self-knowledge of women.

Finally, we compare the number and the welfare of marriages of the PSEI with those of the T3E. As we show in Example 1, the PSEI and the T3E hold under some parameter ranges.

Example 2 *Let us assume that* $\lambda_k^i = \frac{1}{3}$, $(i = m, w,\ k = H, M, L)$. *At this time, the number of marriages for the PSEI and the T3E are* $T^p = \frac{2}{9} N\alpha$ *and* $T^{T3} = \frac{47}{189} N\alpha$, *respectively, from (28) and (30). Then,* $T^{T3} > T^p$. *The welfare of marriages for the PSEI and the T3E are* $W^p = \frac{1}{9} N\alpha(2x_H + x_L + x_M)$ *and* $W^{T3} = \frac{1}{189} N\alpha(42x_H + 31x_L + 21x_M)$ *respectively, from (32) and (34). Hence,* $W^{T3} > W^p$.

When $3\alpha > 2r$ and $F_m(x)$ and $F_w(x)$ are discrete uniform distributions, multiple equilibria arise. At this time, the PSEI and the T3E are not Pareto-rankable: apparently underconfident women prefer the PSEI, and low-type men prefer the T3E. However, the welfare of marriages of the T3E is raised relative to that of the PSEI as a whole. Since apparently underconfident women accept middle- and low-type men in the T3E, the number of marriages of the T3E is raised relative to that of the PSEI. As a result, the welfare of the T3E is also raised relative to that of the PSEI.

5 Discussion

Two types In this paper, we assume not two but three types of agents in order to show the influence of the indirect externality of apparent overconfidence on the market. If we consider two types of agents in the case of apparent overconfidence, the indirect externality does not occur, and we cannot find a case in which the indirect externality of apparent overconfidence prevents the lowest type of agents from marrying. To see this, now, let us assume two types of agents: good and bad. Let us assume that, when all agents have perfect self-knowledge, the PSE occurs. To describe the apparent overconfidence, let us assume that i_0-type women $(i = g, b)$ reject bad-type men. Therefore, there are two kinds of bad-type women with respect to differences in their beliefs: bad-type women who are apparently overconfident and bad-type women who know their own types. However, good-type men do not change their reservation utility levels relative to the case of perfect self-knowledge because all women want to marry them. Then, the indirect externality of apparent overconfidence does not occur. Hence, bad-type men always marry bad-type women with perfect self-knowledge.

On the other hand, in the case of apparent underconfidence with two types of agents, we obtain, qualitatively, the same result as that of Proposition 4. That is, the indirect externality of apparent underconfidence does not occur, and, as a result, all agents can then marry.

Two-sided imperfect self-knowledge In this paper, we assume a one-sided imperfect self-knowledge: none of the women initially know their own type, whereas all men know their own types. This one-sided imperfect self-knowledge assumption is important in order to clarify the influence of imperfect self-knowledge. From Lemma 2, the uncertainty of an agent's own type affects her own expected life utility. Moreover, the existence of others with imperfect self-knowledge also affects agents' expected life utilities from Lemma 1. We can analyze these two influences on the expected life utility of an agent separately, under the assumption of the one-sided imperfect self-knowledge. The one-sided imperfect self-knowledge assumption describes the following situations: in the context of the labor market, a firm will have more information about its own type than a worker, since a firm has more experience than a worker. In the context of the marriage market, when more men work outside the home than women, it will be easier for men than for women to get the objective data on their own charm, such as income, position at work, and social status.

Although two-sided imperfect self-knowledge — all men and women initially lack knowledge of their own types — is a nontrivial extension, our results suggest that, if two-sided imperfect self-knowledge is assumed in the apparent overconfidence case, the reservation level of any agent ('he') will be affected by the following two factors: (i) the large proportion of apparently overconfident women who now reject his type and (ii) the uncertainty of his own type. The first element always lowers his reservation level from Lemma 1. For the second element, as we show in Lemma 2, his reservation level is lowered or raised relative to that in the case of perfect self-knowledge. Therefore, we cannot analyze the influence of these two factors on the reservation level of an agent separately. Hence, two-sided imperfect self-knowledge will make the analysis more complex.[45] Such work is left for future research.

The assumption of apparent over- or underconfidence We adopt the assumption that a k_0-type woman rejects a middle-type man in the apparent overconfidence case because we focus on the case in which *middle*-type women are apparently overconfident in order to show that the indirect externality of apparent overconfidence affects the marriage behavior of lower-type agents.

If k_0-type women accept middle-type men (or reject low-type men), all agents can marry. To see this, let us assume that k_0-type women accept middle-type men. At this time, L_0-type women are apparently overconfident and H_0-type women are apparently underconfident. The apparent underconfidence of H_0-type women does not change the behavior of middle-type men for the same reason as in Lemma 4 (for details, see the Appendix B). Moreover, the indirect externality of apparent overconfidence by low-type women does not arise similarly to that in two types of agents.

Let us consider another apparent overconfidence case in which M_{HM}-type women reject middle-type men. However, at this time, the indirect externality of apparent overconfidence does not arise. It is noteworthy that an M_0-type woman becomes M_{HM}-type woman by the offer from a middle- or low- type man who rejects a low-type woman (now, we assume that high-type men reject middle-type women). If a middle-type man accepts a low-type woman due to the existence of M_{HM}-type women, low-type men also accept low-type women. However, this contradicts the assumption that a middle- or a low-type man rejects a low-type woman.

On the other hand, in the case of apparent underconfidence, let us assume that an M_{ML}-type woman accepts a low-type man. Other than this assumption, we can consider the following cases: (a) a k_0-type woman accepts a low-type man and (b) a k_0-type woman accepts a middle-type man (or rejects a low-type man). However, we qualitatively obtain the same results as in Lemma 4 and Proposition 4 (see the Appendix B for the case in which k_0-type women accept middle-type men). Therefore, we adopt the assumption that an M_{ML}-type woman accepts a low-type man in the case of apparent underconfidence in order to use the framework of analysis in the PSEI again.

Benchmark case In our analysis, we consider the case in which $x_M < R_H^*$ and $x_L < R_M^*$ hold as the benchmark case. If we consider the case in which $x_M \geq R_H^*$ and $x_L < R_M^*$ hold as the benchmark case, the result is the same as the case of the two types. That is, the indirect externality of apparent overconfidence does not occur. To see this, let us define the next

[45] In the case of apparent underconfidence, only the uncertainty of his own type will affect his reservation level.

situation as a benchmark case: if all agents know their own types perfectly, $x_M \geq R_H^*$ and $x_L < R_M^*$ hold. Let us assume that, under $x_M \geq R_H^*$ and $x_L < R_M^*$, k_0-type women reject middle-type men, and then middle-type men accept low-type women due to the rejection from M_0-type women. That is, the indirect externality of apparent overconfidence occurs. However, the reservation utility of a k_0-type woman is always lower than R_H^*, as she assigns probabilities to her own types, similarly to Lemma 2. This contradicts the assumption that under $x_M \geq R_H^*$ and $x_L < R_M^*$, k_0-type women reject middle-type men. Therefore, when $x_M \geq R_H^*$ and $x_L < R_M^*$, k_0-type women always accept middle-type men.[46]

Under $x_M < R_H^*$ and $x_L \geq R_M^*$, the indirect externality of apparent overconfidence does not occur. When $x_M < R_H^*$ and $x_L \geq R_M^*$, there are few enough middle-type agents. Therefore, even if there are some middle-type women who reject middle-type men due to imperfect self-knowledge, middle-type men do not change their behavior: they accept low-type women. Now, as there are few enough middle-type men ($x_L \geq R_M^*$), some low-type women (at least, the low-type women who were rejected by high-type men) always accept low-type men. Therefore, in this case, the indirect externality of apparent overconfidence does not occur.[47]

6 Concluding remarks

We analyze a two-sided search model in which we presume that all women initially do not know their own type and they then learn their own type from the offers or rejections by men. With this belief-updating process, the two-sided aspect of search problem generates a significant interest. Only agents of the same type marry when all agents know their own type perfectly. However, if women who are unsure of their own type reject the men whom they accept when they know their own type, and, if there are enough of these women in the market, the lowest type men can never marry. On the other hand, if women with imperfect self-knowledge accept men whom the women with perfect self-knowledge reject, there are no agents who can never marry.

We conclude with a discussion of some possible further extensions of this model. First, this paper assumes that there is no divorce. However, when a woman marries a man before thoroughly understanding her own type, she may learn about her actual type after she gets married. In this case, the divorce rate and the marriage rate will be influenced by this learning

[46]The assumption of $x_M \geq R_H^*$ means that there are a few high-type men and women in the market. If $x_M \geq R_H^*$ and $x_L < R_M^*$, any high-type woman accepts a middle-type man. In this case, even if high-type women with imperfect self-knowledge accept middle-type men, the behavior of these women is the same as that of the high-type with perfect self-knowledge. Then, a middle-type man does not change his behavior: he accepts a middle-type woman. If some middle-type women with imperfect self-knowledge accept low-type men under $x_M \geq R_H^*$ and $x_L < R_M^*$, the indirect externality of apparent underestimation does not occur in the steady state for the same reason as in Lemma 4 qualitatively.

[47]When $x_M < R_H^*$ and $x_L \geq R_M^*$, the following cases of apparent underconfidence can be considered. If high-type women with imperfect knowledge accept middle-type men under $x_M < R_H^*$ and $x_L \geq R_M^*$, the indirect externality of apparent underconfidence does not occur for the same reason as Lemma 4. If middle-type women with imperfect knowledge accept low-type men under $x_M < R_H^*$ and $x_L \geq R_M^*$, low-type men do not change their behavior: they accept low-type women. Then, in this case, there is no influence of the apparent underconfidence of middle-type women.

in marriage. Next, we assume three types of agent. If we consider a model in which there are n type agents and there are many clusters of marriages, the learning process about one's own type will be more complex. However, if there are n type agents and three clusters of marriages are generated by large enough α, our results also apply to this case.

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Appendix A

Proof of Lemma 1: The reservation utility of a middle-type man for a low-type woman can be calculated as follows: now, a fraction $\eta \in (0,1)$ of middle-type women and a fraction $\zeta \in (0,1)$ of low-type women accept middle-type men. If a middle-type man turns down a low-type woman ($V^r_{Mm} > x_L/r$), his value function becomes

$$rV^r_{Mm} = \alpha\eta\lambda^w_M \left(\frac{x_M}{r} - V^r_{Mm}\right),$$

where the first subscript of V denotes an agent's type and the second subscript of V means a "man."

Conversely, when a middle-type man proposes to a low-type woman ($V^a_{Mm} \leq x_L/r$),

$$rV^a_{Mm} = \alpha\eta\lambda^w_M \left(\frac{x_M}{r} - V^a_{Mm}\right) + \alpha\zeta\lambda^w_L \left(\frac{x_L}{r} - V^a_{Mm}\right).$$

Hence, we have his reservation utility level for declining a low-type woman, $\frac{\alpha\eta\lambda^w_M x_M}{\left(r+\alpha\eta\lambda^w_M\right)} \equiv R^p_{Mm}$. Compared to the PSE, we have

$$R^p_{Mm} - R^*_M = -\frac{r\alpha\lambda^w_M x_M(1-\eta)}{\left(r+\alpha\lambda^w_M\right)\left(r+\alpha\eta\lambda^w_M\right)} < 0.$$

■

Proof of Lemma 2: An l_{ML}-type woman thinks that her actual type is a middle-type with probability $\frac{\mu\lambda^w_M}{\mu\lambda^w_M+\psi(1-\nu)\lambda^w_L}$ and a low-type with probability $\frac{\psi(1-\nu)\lambda^w_L}{\mu\lambda^w_M+\psi(1-\nu)\lambda^w_L}$. The reservation utility of an l_{ML}-type woman for a low-type man can be calculated as follows: if an l_{ML}-type woman turns down a low-type man ($V^r_{l_{ML}} > x_L/r$), her value function becomes

$$r\hat{V}^r_{l_{ML}} = \frac{\mu\lambda^w_M}{\mu\lambda^w_M+\psi(1-\nu)\lambda^w_L}\left[\alpha\lambda^m_M\left(\frac{x_M}{r} - V^r_{M_{ML}}\right)\right] + \frac{\psi(1-\nu)\lambda^w_L}{\mu\lambda^w_M+\psi(1-\nu)\lambda^w_L}\left[\alpha\lambda^m_M\left(V_{L_L} - V^r_{L_{ML}}\right)\right] \quad (35)$$

$$rV_{L_L} = \alpha\lambda^m_L\left(\frac{x_L}{r} - V_{L_L}\right), \quad (36)$$

where $rV^r_{l_{ML}} \equiv \frac{\mu\lambda^w_M rV^r_{M_{ML}} + \psi(1-\nu)\lambda^w_L rV^r_{L_{ML}}}{\mu\lambda^w_M+\psi(1-\nu)\lambda^w_L}$. The second term in Equation (35) means that if the actual type of an l_{ML}-type woman is a low-type, she learns that she is a low-type by meeting a middle-type man. Then, she changes her value function to (36), since she is accepted by a low-type man.

When an l_{ML}-type woman accepts a low-type man, her value function is

$$\begin{aligned} r\hat{V}^a_{l_{ML}} \quad = \quad & \frac{\mu\lambda^w_M}{\mu\lambda^w_M+\psi(1-\nu)\lambda^w_L}\left[\alpha\lambda^m_M\left(\frac{x_M}{r} - V^a_{M_{ML}}\right) + \alpha\lambda^m_L\left(\frac{x_L}{r} - V^a_{M_{ML}}\right)\right] \\ & + \frac{\psi(1-\nu)\lambda^w_L}{\mu\lambda^w_M+\psi(1-\nu)\lambda^w_L}\left[\alpha\lambda^m_M\left(V_{L_L} - V^a_{L_{ML}}\right) + \alpha\lambda^m_L\left(\frac{x_L}{r} - V^a_{L_{ML}}\right)\right], \quad (37) \end{aligned}$$

where $rV^a_{l_{ML}} \equiv \frac{\mu\lambda^w_M rV^a_{M_{ML}} + \psi(1-\nu)\lambda^w_L rV^a_{L_{ML}}}{\mu\lambda^w_M+\psi(1-\nu)\lambda^w_L}$. From (35)-(37), the reservation utility of an l_{ML}-

type woman for a low-type man is

$$R^p_{l_{ML}} \equiv \frac{\lambda^w_M \lambda^m_M \alpha\mu x_M \left(r+\alpha\lambda^m_L\right)}{\lambda^w_M \mu\left(r+\alpha\lambda^m_L\right)\left(r+\alpha\lambda^m_M\right)+\lambda^w_L r\psi(1-\nu)\left(r+\alpha\lambda^m_L+\alpha\lambda^m_M\right)}.$$

Compared to the benchmark case, we have

$$R^p_{l_{ML}} - R^*_M = -\frac{x_M\left(r+\alpha\lambda^m_L+\alpha\lambda^m_M\right)r\lambda^w_L\alpha\psi(1-\nu)\lambda^m_M x}{\left(r+\alpha\lambda^m_M\right)\left(\lambda^w_M\mu\left(r+\alpha\lambda^m_L\right)\left(r+\alpha\lambda^m_M\right)+\lambda^w_L r\psi(1-\nu)\left(r+\alpha\lambda^m_L+\alpha\lambda^m_M\right)\right)} < 0.$$

On the other hand, the reservation utility of a j_{HM}-type woman for a middle-type man can be calculated as follows: if a j_{HM}-type woman turns down a middle-type man ($V^r_{j_{HM}} > x_M/r$), her value function becomes

$$r\hat{V}^r_{j_{HM}} = \frac{\theta\lambda^w_H}{\theta\lambda^w_H+\gamma(1-\phi)\lambda^w_M}\left[\alpha\lambda^m_H\left(\frac{x_H}{r} - V^r_{H_{HM}}\right)\right] + \frac{\gamma(1-\phi)\lambda^w_M}{\theta\lambda^w_H+\gamma(1-\phi)\lambda^w_M}\left[\alpha\lambda^m_H\left(V_{M_M} - V^r_{M_{HM}}\right)\right] \quad (38)$$

$$rV_{M_M} = \alpha\lambda^m_M\left(\frac{x_M}{r} - V_{M_M}\right), \quad (39)$$

where $r\hat{V}^r_{j_{HM}} = \frac{\theta\lambda^w_H rV^r_{H_{HM}}}{\theta\lambda^w_H+\gamma(1-\phi)\lambda^w_M} + \frac{\gamma(1-\phi)\lambda^w_M rV^r_{M_{HM}}}{\theta\lambda^w_H+\gamma(1-\phi)\lambda^w_M}$. The first term in the second square bracket in (38) implies that, if the actual type of a j_{HM}-type woman is the middle-type, she learns that she is a middle-type after a meeting with a high-type man. She then changes her value function to (39) as middle-type men accept middle-type women.

When she accepts a middle-type man, her value function is

$$\begin{aligned} r\hat{V}^a_{j_{HM}} &= \frac{\theta\lambda^w_H}{\theta\lambda^w_H+\gamma(1-\phi)\lambda^w_M}\left[\alpha\lambda^m_H\left(\frac{x_H}{r} - V^a_{H_{HM}}\right) + \alpha\lambda^m_M\left(\frac{x_M}{r} - V^a_{H_{HM}}\right)\right] \\ &\quad + \frac{\gamma(1-\phi)\lambda^w_M}{\theta\lambda^w_H+\gamma(1-\phi)\lambda^w_M}\left[\alpha\lambda^m_H\left(V_{i_M} - V^a_{M_{HM}}\right) + \alpha\lambda^m_M\left(\frac{x_M}{r} - V^a_{M_{HM}}\right)\right], \end{aligned} \quad (40)$$

where $r\hat{V}^a_{j_{HM}} = \frac{\theta\lambda^w_H rV^a_{H_{HM}}}{\theta\lambda^w_H+\gamma(1-\phi)\lambda^w_M} + \frac{\gamma(1-\phi)\lambda^w_M rV^a_{M_{HM}}}{\theta\lambda^w_H+\gamma(1-\phi)\lambda^w_M}$. Therefore, the reservation utility of a j_{HM}-type woman for a middle-type man is

$$R^p_{j_{HM}} \equiv \frac{\lambda^w_H\lambda^m_H\theta\alpha x_H\left(r+\alpha\lambda^m_M\right)}{\left(\lambda^w_H\theta\left(r+\alpha\lambda^m_H\right)\left(r+\alpha\lambda^m_M\right)+r\gamma(1-\phi)\lambda^w_M\left(r+\alpha\lambda^m_H+\alpha\lambda^m_M\right)\right)}.$$

Compared to the benchmark case, we have

$$R^p_{j_{HM}} - R^*_H = -\frac{r\alpha\gamma(1-\phi)\lambda^m_H\lambda^w_M x_H\left(r+\alpha\lambda^m_H+\alpha\lambda^m_M\right)}{\left(r+\alpha\lambda^m_H\right)\left(\lambda^w_H\theta\left(r+\alpha\lambda^m_H\right)\left(r+\alpha\lambda^m_M\right)+r\gamma(1-\phi)\lambda^w_M\left(r+\alpha\lambda^m_H+\alpha\lambda^m_M\right)\right)} < 0.$$

Given the behavior of all agents except k_0-type women, we can obtain the lifetime utility of a k_0-type woman. If she rejects a middle-type man, her value function is

$$\begin{aligned} r\hat{V}^r &= \frac{(1-\theta)\lambda^w_H}{(1-\theta)\lambda^w_H+(1-\mu-\gamma)\lambda^w_M+(1-\psi-\kappa)\lambda^w_L}\left[\alpha\lambda^m_H\left(\frac{x_H}{r} - V^r_H\right) + \alpha\lambda^m_M\left(\hat{V}_{j_{HM}} - V^r_H\right)\right] \\ &\quad + \frac{(1-\mu-\gamma)\lambda^w_M}{(1-\theta)\lambda^w_H+(1-\mu-\gamma)\lambda^w_M+(1-\psi-\kappa)\lambda^w_L}\left[\alpha\lambda^m_H\left(\hat{V}_{l_{ML}} - V^r_M\right) + \alpha\lambda^m_M\left(\hat{V}_{j_{HM}} - V^r_M\right)\right] \\ &\quad + \frac{(1-\psi-\kappa)\lambda^w_L}{(1-\theta)\lambda^w_H+(1-\mu-\gamma)\lambda^w_M+(1-\psi-\kappa)\lambda^w_L}\left[\alpha\lambda^m_H\left(\hat{V}_{l_{ML}} - V^r_L\right) + \alpha\lambda^m_M\left(V_{L_L} - V^r_L\right)\right], \end{aligned} \quad (41)$$

where $rV^r \equiv \frac{(1-\theta)\lambda^w_H rV^r_H+(1-\mu-\gamma)\lambda^w_M rV^r_M+(1-\psi-\kappa)\lambda^w_L rV^r_L}{(1-\theta)\lambda^w_H+(1-\mu-\gamma)\lambda^w_M+(1-\psi-\kappa)\lambda^w_L}$. The second term in the third square bracket in (41) implies that, if the actual type of a k_0-type woman is the low-type, she learns that she is the low-type after meeting a middle-type man. She then changes her value function to (36) as a low-type man accepts a low-type woman. The second term in the first (or the second) square bracket in Equation (41) means that, if the actual type of a k_0-type woman is the high- (or middle-) type, she learns that she is the high- or the middle-type after meeting a middle-type man. She then changes her optimal strategy to $R^p_{j_{HM}}$. Likewise, the first term in the second (or the third) square bracket in Equation (41) means that, if the actual type of a k_0-type woman is the middle- (or low-) type, she learns that she is the middle- or the low-type after meeting a high-type man. She then changes her optimal strategy to $R^p_{l_{ML}}$.

If a k_0-type woman accepts a middle-type man, her value function becomes

$$\begin{aligned} r\hat{V}^a &= \tfrac{(1-\theta)\lambda_H^w}{(1-\theta)\lambda_H^w+(1-\mu-\gamma)\lambda_M^w+(1-\psi-\kappa)\lambda_L^w}\left[\alpha\lambda_H^m\left(\frac{x_H}{r}-V_H^a\right)+\alpha\lambda_M^m\left(\frac{x_M}{r}-V_H^a\right)\right] \\ &\quad+\tfrac{(1-\mu-\gamma)\lambda_M^w}{(1-\theta)\lambda_H^w+(1-\mu-\gamma)\lambda_M^w+(1-\psi-\kappa)\lambda_L^w}\left[\alpha\lambda_H^m\left(\hat{V}_{l_{ML}}-V_M^a\right)+\alpha\lambda_M^m\left(\frac{x_M}{r}-V_M^a\right)\right] \\ &\quad+\tfrac{(1-\psi-\kappa)\lambda_L^w}{(1-\theta)\lambda_H^w+(1-\mu-\gamma)\lambda_M^w+(1-\psi-\kappa)\lambda_L^w}\left[\alpha\lambda_H^m\left(\hat{V}_{l_{ML}}-V_L^a\right)+\alpha\lambda_M^m\left(V_{L_L}-V_L^a\right)\right], \end{aligned}$$

where $rV^a \equiv \frac{(1-\theta)\lambda_H^w rV_H^a+(1-\mu-\gamma)\lambda_M^w rV_M^a+(1-\psi-\kappa)\lambda_L^w rV_L^a}{(1-\theta)\lambda_H^w+(1-\mu-\gamma)\lambda_M^w+(1-\psi-\kappa)\lambda_L^w}$. Therefore, the reservation utility of a k_0-type woman for a middle-type man is

$$R^p_{k_0} \equiv \frac{\lambda_H^w\lambda_H^m\theta\alpha x_H\left(r+\alpha\lambda_M^m\right)}{\left(\theta\lambda_H^w\left(r+\alpha\lambda_H^m\right)\left(r+\alpha\lambda_M^m\right)+r\gamma\lambda_M^w(1-\phi)\left(r+\alpha\lambda_H^m+\alpha\lambda_M^m\right)\right)} = R^p_{j_{HM}} > (\leq)\, x_M.$$

If $R^p_{k_0} > (\leq)\, x_M$, a k_0-type woman rejects (accepts) a middle-type man. ■

Proof of Propostition 2: From (8) - (12), we obtain

$$\theta = \gamma = \frac{\lambda_M^m}{\lambda_H^m+\lambda_M^m}, \tag{42}$$

$$\mu = \frac{\left(\lambda_H^m\right)^2}{\left(\lambda_H^m+\lambda_M^m\right)^2},\ \phi = \psi = \frac{\lambda_H^m}{\left(\lambda_H^m+\lambda_M^m\right)}, \tag{43}$$

$$\nu = \frac{\lambda_M^m}{\left(\lambda_L^m+\lambda_M^m\right)},\ \kappa = \frac{\left(\lambda_M^m\right)^2}{\left(\lambda_L^m+\lambda_M^m\right)\left(\lambda_H^m+\lambda_M^m\right)}. \tag{44}$$

The proposition follows immediately by substituting (42)-(44) into (5), (6) and (7). ■

Proof of Lemma 3: Since a middle-type man accepts a low-type woman, an M_{ML}-type woman and an L_{ML}-type woman face the same problem. They decide whether to accept or reject low-type men. Therefore, if an l_{ML}-type ($l = M, L$) woman rejects a low-type man, her value function is

$$r\hat{V}^r_{l_{ML}} = \alpha\lambda_M^m\left(\frac{x_M}{r}-\hat{V}^r_{l_{ML}}\right). \tag{45}$$

When she accepts a low-type man, her value function is

$$r\hat{V}^a_{l_{ML}} = \alpha\lambda_M^m\left(\frac{x_M}{r}-\hat{V}^a_{l_{ML}}\right)+\alpha\lambda_L^m\left(\frac{x_L}{r}-\hat{V}^a_{l_{ML}}\right). \tag{46}$$

Therefore, the reservation utility of an l_{ML}-type woman for a low-type man is

$$R_{l_{ML}}^{T1} \equiv \frac{\alpha\lambda_M^m x_M}{r+\alpha\lambda_M^m} = R_M^*.$$

As $x_L < R_M^*$, $x_L < R_{l_{ML}}$ holds. Hence, an l_{ML}-type woman turns down a low-type man.

Given the behavior of all agents, except a k_0-type woman, we can obtain the lifetime utility of a k_0-type ($k = H, M, L$) woman. If she rejects a middle-type man, her value function is

$$r\hat{V}^r = \frac{\lambda_H^w}{\lambda_H^w+(1-\tau)\left(\lambda_M^w+\lambda_L^w\right)}\alpha\lambda_H^m\left(\frac{x_H}{r}-V_H^r\right)+\frac{(1-\tau)\left(\lambda_M^w+\lambda_L^w\right)}{\lambda_H^w+(1-\tau)\left(\lambda_M^w+\lambda_L^w\right)}\alpha\lambda_H^m\left(\hat{V}_{l_{ML}}-V_M^r\right), \quad (47)$$

where $rV^r \equiv \frac{\lambda_H^w rV_H^r}{\lambda_H^w+(1-\tau)\left(\lambda_M^w+\lambda_L^w\right)}+\frac{(1-\tau)r\left(\lambda_M^w V_M^r+\lambda_L^w V_L^r\right)}{\lambda_H^w+(1-\tau)\left(\lambda_M^w+\lambda_L^w\right)}$. The second term in Equation (47) means that, if the actual type of a k_0-type woman is the middle- or low-type, she becomes an l_{ML}-type woman after meeting a high-type man. She then changes her optimal strategy to $R_{l_{ML}}^{T1}$.

If a k_0-type woman accepts a middle-type man, her value function becomes

$$\begin{aligned} r\hat{V}^a \quad = \quad & \frac{\lambda_H^w}{\lambda_H^w+(1-\tau)\left(\lambda_M^w+\lambda_L^w\right)}\left(\alpha\lambda_H^m\left(\frac{x_H}{r}-V_H^a\right)+\alpha\lambda_M^m\left(\frac{x_M}{r}-V_H^a\right)\right) \\ & +\frac{(1-\tau)\left(\lambda_M^w+\lambda_L^w\right)}{\lambda_H^w+(1-\tau)\left(\lambda_M^w+\lambda_L^w\right)}\left(\alpha\lambda_H^m\left(\hat{V}_{l_{ML}}-V_M^a\right)+\alpha\lambda_M^m\left(\frac{x_M}{r}-V_M^a\right)\right), \end{aligned}$$

where $r\hat{V}^a \equiv \frac{\lambda_H^w rV_H^a}{\lambda_H^w+(1-\tau)\left(\lambda_M^w+\lambda_L^w\right)}+\frac{(1-\tau)r\left(\lambda_M^w V_M^a+\lambda_L^w V_L^a\right)}{\lambda_H^w+(1-\tau)\left(\lambda_M^w+\lambda_L^w\right)}$. Therefore, the reservation utility of a k_0-type woman for a middle-type man is

$$R_{k_0}^{T1} \equiv \frac{\lambda_H^w\lambda_H^m\alpha x_H\left(r+\alpha\lambda_M^m\right)}{\left(\lambda_H^w\left(r+\alpha\lambda_M^m\right)\left(r+\alpha\lambda_H^m\right)+r(1-\tau)\left(\lambda_L^w+\lambda_M^w\right)\left(r+\alpha\lambda_H^m+\alpha\lambda_M^m\right)\right)}.$$

Compared to the benchmark case, we have

$$R_{k_0}^{T1}-R_H^* = -\frac{\left(r(1-\tau)\left(\lambda_L^w+\lambda_M^w\right)+\alpha(1-\tau)\left(\lambda_L^w+\lambda_M^w\right)\left(\lambda_H^m+\lambda_M^m\right)\right)r\alpha\lambda_H^m x_H}{\left(r+\alpha\lambda_H^m\right)\left(\lambda_H^w\left(r+\alpha\lambda_M^m\right)\left(r+\alpha\lambda_H^m\right)+r(1-\tau)\left(\lambda_L^w+\lambda_M^w\right)\left(r+\alpha\lambda_H^m+\alpha\lambda_M^m\right)\right)} < 0.$$

■

Proof of Proposition 3: From (15)-(16), we obtain $\tau = \varpi = \frac{\lambda_H^m}{\lambda_H^m+\lambda_M^m}$. The proposition follows immediately by substituting $\tau = \varpi = \frac{\lambda_H^m}{\lambda_H^m+\lambda_M^m}$ into (14) and (13). ■

Proof of Lemma 4: Now we consider the T2EC in which apparently underconfident M_{ML}-type women accept low-type men. Thus, a low-type man rejects a low-type woman ($R_{Lm}^{T2} > x_L$) with the expectation to marry an M_{ML}-type woman. In this case, the decision of an l_{ML}-type ($l = M, L$) woman is as follows: by meeting a high-type man, an l_{ML}-type woman thinks that her actual type is the middle-type with probability $\frac{\mu_2\lambda_M^w}{\mu_2\lambda_M^w+\psi_2\lambda_L^w}$ and the low-type with probability $\frac{\psi_2\lambda_L^w}{\mu_2\lambda_M^w+\psi_2\lambda_L^w}$. Now, the option of an l_{ML}-type woman is to marry or to reject a low-type man. If she accepts a low-type man, her value function is

$$\begin{aligned}
\hat{V}^a_{l_{ML}} &= \tfrac{\mu_2\lambda^w_M}{\mu_2\lambda^w_M+\psi_2\lambda^w_L}\left[\alpha\lambda^m_M\left(\frac{x_M}{r}-V^a_{M_{ML}}\right)+\alpha\lambda^m_L\left(\frac{x_L}{r}-V^a_{M_{ML}}\right)\right] \\
&\quad +\tfrac{\psi_2\lambda^w_L}{\mu_2\lambda^w_M+\psi_2\lambda^w_L}\left[\alpha\left(\lambda^m_M+\lambda^m_L\right)\left(0-V^a_{L_{ML}}\right)\right],
\end{aligned}$$

where $r\hat{V}^a_{l_{ML}} \equiv \frac{\mu_2\lambda^w_M}{\mu_2\lambda^w_M+\psi_2\lambda^w_L}rV^a_{M_{ML}} + \frac{\psi_2\lambda^w_L}{\mu_2\lambda^w_M+\psi_2\lambda^w_L}rV^a_{L_{ML}}$.

If she turns down a low-type man

$$\begin{aligned}
r\hat{V}^r_{l_{ML}} &= \tfrac{\mu_2\lambda^w_M}{\mu_2\lambda^w_M+\psi_2\lambda^w_L}\left[\alpha\lambda^m_M\left(\frac{x_M}{r}-V^r_{M_{ML}}\right)+\alpha\lambda^m_L\left(V_{M_M}-V^r_{M_{ML}}\right)\right] \\
&\quad +\tfrac{\psi_2\lambda^w_L}{\mu_2\lambda^w_M+\psi_2\lambda^w_L}\left[\alpha\left(\lambda^m_M+\lambda^m_L\right)\left(0-V^r_{L_{ML}}\right)\right], \qquad (48) \\
rV_{M_M} &= \alpha\lambda^m_M\left(\frac{x_M}{r}-V_{M_M}\right),
\end{aligned}$$

where $r\hat{V}^r_{l_{ML}} \equiv \frac{\mu_2\lambda^w_M}{\mu_2\lambda^w_M+\psi_2\lambda^w_L}rV^r_{M_{ML}} + \frac{\psi_2\lambda^w_L}{\mu_2\lambda^w_M+\psi_2\lambda^w_L}rV^r_{L_{ML}}$. The second term in the first square brackets in Equation (48) means that, if the actual type of an l_{ML}-type woman is the middle-type, she learns that she is the middle-type after meeting a low-type man. Here, $\hat{V}^a_{l_{ML}} < \hat{V}^r_{l_{ML}}$ holds, as $R^*_M > x_L$. That is, an l_{ML}-type ($l = M, L$) woman always rejects a low-type man. This contradicts the assumption that an l_{ML}-type woman accepts a low-type man. Therefore, the T2EC is not an equilibrium.[48] ■

Proof of Lemma 5: As $R^{T3}_{Mm} > x_L \geq R^{T3}_{Lm}$, the information about types of women from the proposals by men and the belief-updating processes of women are the same as those in the PSEI. Hence, the reservation utility levels of women are obtained in the same manner as those in the PSEI:

$$\begin{aligned}
R^{T3}_{l_{ML}} &\equiv \tfrac{\lambda^w_M\lambda^m_M\alpha\mu_2 x_M\left(r+\alpha\lambda^m_L\right)}{r\psi_2(1-\nu_2)\lambda^w_L\left(r+\alpha\lambda^m_L+\alpha\lambda^m_M\right)+\mu_2\lambda^w_M\left(r+\alpha\lambda^m_L\right)\left(r+\alpha\lambda^m_M\right)}\left(< R^*_M\right), \\
R^{T3}_{j_{HM}} &= R^{T3}_{k_0} \equiv \tfrac{\alpha\theta_2\lambda^w_H\lambda^m_H x_H\left(r+\alpha\lambda^m_M\right)}{\left(r\lambda^w_M\gamma_2(1-\phi_2)\left(r+\alpha\lambda^m_H+\alpha\lambda^m_M\right)+\theta_2\lambda^w_H\left(r+\alpha\lambda^m_H\right)\left(r+\alpha\lambda^m_M\right)\right)}\left(< R^*_H\right).
\end{aligned}$$

■

Proof of Proposition 4: From (21) - (26), we obtain

$$\theta_3 = \tfrac{\lambda^m_M}{\lambda^m_H+\lambda^m_M}, \qquad (49)$$

$$\mu_3 = \tfrac{\left(\lambda^m_H\right)^2}{\lambda^m_H+\left(\lambda^m_M+\lambda^m_L\right)\left(\lambda^m_H+\lambda^m_M\right)}, \qquad (50)$$

$$\gamma_3 = \tfrac{\left(\lambda^m_M+\lambda^m_L\right)\left(\lambda^m_H+\lambda^m_M\right)}{\left(\lambda^m_H+\left(\lambda^m_M+\lambda^m_L\right)\left(\lambda^m_H+\lambda^m_M\right)\right)}, \phi_3 = \tfrac{\lambda^m_H}{\left(\lambda^m_M+\lambda^m_H\right)}, \qquad (51)$$

$$\psi_3 = \lambda^m_H,\ \nu_3 = \tfrac{\lambda^m_M}{\left(\lambda^m_M+\lambda^m_L\right)},\ \kappa_3 = \lambda^m_M. \qquad (52)$$

The proposition follows immediately by substituting (49)-(52) into (17), (18) (19) and (20).■

[48] A j_{HM}- and a k_0-type woman always reject a middle-type man when $x_M < R^*_H,\ x_L < R^{T2}_{Mm} < R^*_M$.

Appendix B

Type 4 equilibrium candidate

In this Appendix, we show that, even if there are many apparently underconfident H_0-type women who accept middle-type men, a middle-type man always accepts a low-type woman.[49] To see this, in this Appendix, we consider a *Type 4 equilibrium candidate* in which high-type agents form the first cluster of marriages, middle-type men and apparently underconfident high-type women, the second cluster, low-type men and middle-type women, the third cluster, and low-type women can never marry. Then, let us assume that a k_0-type woman chooses the strategy to accept a middle-type man.

We first investigate the strategy of men, given the behavior of the women above. A high-type man has the same reservation utility as a high-type man in the benchmark case since all women want to marry high-type men. Since we consider the equilibrium in which a k_0-type woman accepts a middle-type man (or rejects a low-type man), some middle-type women accept a low-type man, let $(1-\theta_4) \in (0,1)$ and $\zeta_4 \in (0,1)$ denote the proportion of H_0-type women who accept middle-type men and that of middle-type women who accept middle-type men, respectively. Furthermore, $\varphi_4 \in (0,1)$ denotes the proportion of middle-type women who accept low-type men. Then, the optimal strategy of a low-type man is obtained in the same manner as in the T2EC, i.e., $R_{Lm}^{T4} \equiv \frac{\alpha\varphi_4\lambda_M^w x_M}{r+\alpha\varphi_4\lambda_M^w}$. On the other hand, the reservation utility of a middle-type man is obtained in the next lemma.

Lemma 6 *Let us assume that $x_M < R_H^*$ and $x_L < R_M^*$ and that $(1-\theta_4) \in (0,1)$ of high-type women and $\zeta_4 \in (0,1)$ of middle-type women accept middle-type men. Furthermore, $\varphi_4 \in (0,1)$ of middle-type women accept low-type men. If*

$$\frac{\alpha\lambda_H^w(1-\theta_4)}{\left(r+\alpha\lambda_H^w(1-\theta_4)\right)} x_H \equiv R_{Mm}^{T4} > (\leq)\ x_M, \tag{53}$$

a middle-type man rejects (accepts) a middle-type woman. In this case, the reservation utility level of a middle-type man increases relative to the benchmark result, i.e., $R_M^* < R_{Mm}^{T4}$.

Proof. First, we consider whether a middle-type man wishes to marry a middle-type woman. If a middle-type man accepts a middle-type woman, i.e., $x_M/r \geq V_{Mm}^a (> x_L/r)$, his discounted lifetime utility is

$$rV_{Mm}^a = \alpha\lambda_H^w (1-\theta_4)\left(\frac{x_H}{r} - V_{Mm}^a\right) + \alpha\zeta_4\lambda_M^w\left(\frac{x_M}{r} - V_{Mm}^a\right). \tag{54}$$

If a middle-type man turns down a middle-type woman ($x_M/r < V_{Mm}^r$), his value function is

$$rV_{Mm}^r = \alpha\lambda_H^w (1-\theta_4)\left(\frac{x_H}{r} - V_{Mm}^r\right).$$

[49] However, the large proportion of H_0-type women in the market increases the possibility that k_0-type women reject middle-type men.

If a middle-type man proposes to a middle-type woman ($x_M/r > V_{Mm} \geq x_L/r$), his value function becomes (54). Hence, from $V_M^r > V_M^a$,

$$\frac{\alpha\lambda_H^w(1-\theta_4)}{\left(r+\alpha\lambda_H^w(1-\theta_4)\right)}x_H \equiv R_{Mm}^{T4} > x_M.$$

If $x_M \geq R_{Mm}^{T4}$, a middle-type man accepts a middle-type woman. ■

This lemma means that, if some high-type women accept middle-type men, a middle-type man's reservation utility level increases relative to that of the benchmark result. This is because a middle-type man expects to marry a high-type woman who accepts a middle-type man. In the following analysis, we assume that $R_{Mm}^{T4} > x_M$ and $R_{Lm}^{T4} > x_L$ as we consider the T4E.

Next, we investigate the strategies of women in the market given $R_{Mm}^{T4} > x_M$, $R_{Lm}^{T4} > x_L$. Now, the means of the proposal and rejection by a high-type man are the same as those in PSEI. Now, the rejection from a middle-type man means that the woman who is rejected by him is either a middle- or a low-type. The offer from a middle-type man means that the woman who is accepted by him is a high-type. The rejection from a low-type man means that a woman who is rejected by him is a low-type. The offer from a low-type man means that the woman who is accepted by him is either a high- or a middle-type.

Therefore, a woman's learning process becomes the following: an H_0-type woman learns that she is a high-type when she is accepted by a high- or a middle-type man. It is noteworthy that she marries a middle-type man, since they propose to each other simultaneously. Another H_0-type woman also learns that she is a high- or a middle-type by meeting a low-type man (we refer to her as 'H_{HM}-type'). An H_{HM}-type woman learns further that she is a high-type after meeting a middle-type man (that is, she becomes the 'H_H-type'). Likewise, an M_0-type woman becomes an M_{HM}-type after she meets a low-type man. An M_{HM}-type woman becomes an M_M-type by meeting a high- or a middle-type man. On the other hand, another M_0-type woman also learns that she is a middle- or a low-type after meeting a high- or a middle-type man (we refer to her as 'M_{ML}-type'). An M_{ML}-type woman leaves the market before recognizing her actual type when she marries a low-type man. An L_0-type woman becomes an L_{ML}-type woman after she meets a high- or a middle-type man. An L_{ML}-type woman learns that she is the low-type by meeting a low-type man. Another L_0-type woman also learns that she is the low-type by meeting a low-type man. A low-type woman who learns that she is the low-type leaves the market as she can never marry.[50]

Therefore, in the market, there are nine kinds of women according to different beliefs: k_0-type women ($k = H, M, L$), l_{ML}-type women ($l = M, L$), j_{HM}-type women ($j = H, M$)), H_H-type women, and M_M-type women. Their optimal strategies are obtained in the next lemma.

Lemma 7 *Let us assume that $x_M < R_H^*$, $x_L < R_M^*$, $R_{Mm}^{T4} > x_M$ and $R_{Lm}^{T4} > x_L$ hold. In this case, an l_{ML}($l = M, L$)-type woman always accepts a low-type man. A j_{HM}($j = H, M$)-type*

[50] If a low-type woman learns that she is a low-type and leaves the market, a single l_0-woman enters the market at once from the 'cloning assumption.'

woman rejects a middle-type man.

Proof. By meeting a high- or a middle-type man, an l_{ML}-type woman thinks that her actual type is the middle-type with probability $\frac{\mu_4 \lambda_M^w}{\mu_4 \lambda_M^w + \psi_4 \lambda_L^w}$ and the low-type with probability $\frac{\psi_4 \lambda_L^w}{\mu_4 \lambda_M^w + \psi_4 \lambda_L^w}$. Now, the option of an l_{ML}-type woman is to marry or to reject a low-type man. If she turns down a low-type man, her expected discounted lifetime utility becomes zero. Therefore, an l_{ML}-type always proposes to a low-type man with

$$r\hat{V}_{l_{ML}} = \frac{\mu_4 \lambda_M^w}{\mu_4 \lambda_M^w + \psi_4 \lambda_L^w} \left[\alpha \lambda_L^m \left(\frac{x_L}{r} - V_{M_{ML}} \right) \right] + \frac{\psi_4 \lambda_L^w}{\mu_4 \lambda_M^w + \psi_4 \lambda_L^w} \left[\alpha \lambda_L^m \left(0 - V_{L_{ML}} \right) \right], \tag{55}$$

where $r\hat{V}_{l_{ML}} \equiv \frac{\mu_4 \lambda_M^w r V_{M_{ML}}}{\mu_4 \lambda_M^w + \psi_4 \lambda_L^w} + \frac{\psi_4 \lambda_L^w r V_{L_{ML}}}{\mu_4 \lambda_M^w + \psi_4 \lambda_L^w}$. At this time, the value of an l_{ML}-type woman becomes zero if her actual type is the low-type and if she meets a low-type man.

From $x_M < R_{Mm}^{T4}$ and $x_L < R_{Lm}^{T4}$, the option of a j_{HM}-type woman is to marry or to reject a middle-type man. Let us assume that a j_{HM}-type woman thinks that her actual type is the high-type with probability $\frac{\theta_4(1-\eta_4)\lambda_H^w}{\theta_4(1-\eta_4)\lambda_H^w + \gamma_4(1-\phi_4)\lambda_M^w}$ and the middle-type with probability $\frac{\gamma_4(1-\phi_4)\lambda_M^w}{\theta_4(1-\eta_4)\lambda_H^w + \gamma_4(1-\phi_4)\lambda_M^w}$.

If she accepts a middle-type man, her value function $\hat{V}_{j_{HM}}^a$ becomes

$$\begin{aligned} r\hat{V}_{j_{HM}}^a &= \frac{\theta_4(1-\eta_4)\lambda_H^w}{\theta_4(1-\eta_4)\lambda_H^w + \gamma_4(1-\phi_4)\lambda_M^w} \left[\alpha \lambda_H^m \left(\frac{x_H}{r} - V_{H_{HM}}^a \right) + \alpha \lambda_M^m \left(\frac{x_M}{r} - V_{H_{HM}}^a \right) \right] \\ &\quad + \frac{\gamma_4(1-\phi_4)\lambda_M^w}{\theta_4(1-\eta_4)\lambda_H^w + \gamma_4(1-\phi_4)\lambda_M^w} \left[\alpha \left(\lambda_H^m + \lambda_M^m \right) \left(V_{M_M} - V_{M_{HM}}^a \right) \right], & (56) \\ rV_{M_M} &= \alpha \lambda_L^m \left(\frac{x_L}{r} - V_{M_M} \right), & (57) \end{aligned}$$

where $r\hat{V}_{j_{HM}}^a \equiv \frac{\theta_4(1-\eta_4)\lambda_H^w}{\theta_4(1-\eta_4)\lambda_H^w + \gamma_4(1-\phi_4)\lambda_M^w} rV_{H_{HM}}^a + \frac{\gamma_4(1-\phi_4)\lambda_M^w}{\theta_4(1-\eta_4)\lambda_H^w + \gamma_4(1-\phi_4)\lambda_M^w} rV_{M_{HM}}^a$. The second square bracket in Equation (56) means as follows: if the actual type of a j_{HM}-type woman is the middle-type and she meets a high- or a middle-type man, she is rejected by them. Therefore, she updates her belief about her own type and then changes her value function to (57).

If a j_{HM}-type woman rejects a middle-type man, her value function $\hat{V}_{j_{HM}}^r$ becomes,

$$\begin{aligned} r\hat{V}_{j_{HM}}^r &= \frac{\theta_4(1-\eta_4)\lambda_H^w}{\theta_4(1-\eta_4)\lambda_H^w + \gamma_4(1-\phi_4)\lambda_M^w} \left[\alpha \lambda_H^m \left(\frac{x_H}{r} - V_{H_{HM}}^r \right) + \alpha \lambda_M^m \left(V_{H_H} - V_{H_{HM}}^r \right) \right] \\ &\quad + \frac{\gamma_4(1-\phi_4)\lambda_M^w}{\theta_4(1-\eta_4)\lambda_H^w + \gamma_4(1-\phi_4)\lambda_M^w} \left[\alpha \left(\lambda_H^m + \lambda_M^m \right) \left(V_{M_M} - V_{M_{HM}}^r \right) \right], & (58) \\ rV_{H_H} &= \alpha \lambda_H^m \left(\frac{x_H}{r} - V_{H_H} \right) & (59) \end{aligned}$$

where $r\hat{V}_{j_{HM}}^r \equiv \frac{\theta_4(1-\eta_4)\lambda_H^w}{\theta_4(1-\eta_4)\lambda_H^w + \gamma_4(1-\phi_4)\lambda_M^w} rV_{H_{HM}}^r + \frac{\gamma_4(1-\phi_4)\lambda_M^w}{\theta_4(1-\eta_4)\lambda_H^w + \gamma_4(1-\phi_4)\lambda_M^w} rV_{M_{HM}}^r$. The second term in first square bracket in Equation (58) means the following: if the actual type of a j_{HM}-type woman is the high-type and she meets a middle-type man, she knows that she is a high-type after this meeting. She then changes her value function to (59). Now, the term in the second square bracket of Equation (58) is the same as that of Equation (56), as a high- and a middle-type man reject a middle-type woman.

From Equations (56)-(59) and $x_M < R^*_H$,

$$\hat{V}^a_{j_{HM}} - \hat{V}^r_{j_{HM}} = \lambda^m_M \lambda^w_H \alpha \frac{r x_M - \alpha\lambda^m_H x_H + \alpha\lambda^m_H x_M}{r\left(r+\alpha\lambda^m_H\right)\left(r+\alpha\lambda^m_H+\alpha\lambda^m_M\right)\left(\lambda^w_H+\lambda^w_M\right)} < 0$$

holds. Therefore, a j_{HM}-type woman always rejects a middle-type man. ■

From Lemma 7, an H_{HM}-type woman marries a high-type man when they meet. However, an H_{HM}-type woman rejects a middle-type man when they meet. After their meeting, an H_{HM}-type woman learns that she is a high-type by an offer from a middle-type man, and she then changes the reservation utility level to R^*_H. Therefore, an H_H-type women marry high-type men when they meet. An M_{HM}-type woman also chooses the option of rejection for a middle-type man. However, she learns that she is a middle-type after she meets a high- or a middle-type man, since she is rejected by them. Then, an M_M-type woman marries a low-type man. An M_{ML}-type woman marries a low-type man when they meet. An L_{ML}-type woman learns that she is a low-type and leaves the market after meeting a low-type man.

With these results, we obtain the discounted lifetime utility of a k_0-type woman when $x_M < R^{T4}_{Mm}$ and $R^{T4}_{Lm} > x_L$. As she accepts a middle-type man, her value function becomes,

$$\begin{aligned} r\hat{V}^a = & \frac{(1-\theta_4)\lambda^w_H}{(1-\theta_4)\lambda^w_H+(1-\mu_4-\gamma_4)\lambda^w_M+(1-\psi_4)\lambda^w_L}\left[\alpha\lambda^m_H\left(\frac{x_H}{r}-V^a_H\right)+\alpha\lambda^m_M\left(\frac{x_M}{r}-V^a_H\right)+\alpha\lambda^m_L\left(\hat{V}_{j_{HM}}-V^a_H\right)\right] \\ & + \frac{(1-\mu_4-\gamma_4)\lambda^w_M}{(1-\theta_4)\lambda^w_H+(1-\mu_4-\gamma_4)\lambda^w_M+(1-\psi_4)\lambda^w_L}\left[\alpha\left(\lambda^m_H+\lambda^m_M\right)\left(\hat{V}_{l_{ML}}-V^a_M\right)+\alpha\lambda^m_L\left(\hat{V}_{j_{HM}}-V^a_M\right)\right] \\ & + \frac{(1-\psi_4)\lambda^w_L}{(1-\theta_4)\lambda^w_H+(1-\mu_4-\gamma_4)\lambda^w_M+(1-\psi_4)\lambda^w_L}\left[\alpha\left(\lambda^m_H+\lambda^m_M\right)\left(\hat{V}_{l_{ML}}-V^a_L\right)+\alpha\lambda^m_L\left(0-V^a_L\right)\right], \quad (60) \end{aligned}$$

where $r\hat{V}^a \equiv \frac{(1-\theta_4)\lambda^w_H rV^a_H+(1-\mu_4-\gamma_4)\lambda^w_M rV^a_M+(1-\psi_4)\lambda^w_L rV^a_L}{(1-\theta_4)\lambda^w_H+(1-\mu_4-\gamma_4)\lambda^w_M+(1-\psi_4)\lambda^w_L}$. The third term in the first and second square brackets in Equation (60) means that, if the actual type of a k_0-type woman is the high- or middle-type and she meets a low-type man, she learns that she is a high- or a middle-type. She then changes her value function to (58). The first term in second and third square brackets in Equation (60) means that, if the actual type of a k_0-type woman is the middle- or low-type and she meets a high-type or a middle-type man, she learns that she is a middle- or a low-type. Therefore, she changes her value function to (55).

If she rejects a middle-type man, her value function becomes,

$$\begin{aligned} r\hat{V}^r = & \frac{(1-\theta_4)\lambda^w_H}{(1-\theta_4)\lambda^w_H+(1-\mu_4-\gamma_4)\lambda^w_M+(1-\psi_4)\lambda^w_L}\left[\alpha\lambda^m_H\left(\frac{x_H}{r}-V^r_H\right)+\alpha\lambda^m_M\left(V_{H_H}-V^r_H\right)+\alpha\lambda^m_L\left(\hat{V}_{j_{HM}}-V^r_H\right)\right] \\ & + \frac{(1-\mu_4-\gamma_4)\lambda^w_M}{(1-\theta_4)\lambda^w_H+(1-\mu_4-\gamma_4)\lambda^w_M+(1-\psi_4)\lambda^w_L}\left[\alpha\left(\lambda^m_H+\lambda^m_M\right)\left(\hat{V}_{l_{ML}}-V^r_M\right)+\alpha\lambda^m_L\left(\hat{V}_{j_{HM}}-V^r_M\right)\right] \\ & + \frac{(1-\psi_4)\lambda^w_L}{(1-\theta_4)\lambda^w_H+(1-\mu_4-\gamma_4)\lambda^w_M+(1-\psi_4)\lambda^w_L}\left[\alpha\left(\lambda^m_H+\lambda^m_M\right)\left(\hat{V}_{l_{ML}}-V^r_L\right)+\alpha\lambda^m_L\left(0-V^r_L\right)\right]. \quad (61) \end{aligned}$$

where $r\hat{V}^r \equiv \frac{(1-\theta_4)\lambda^w_H rV^r_H+(1-\mu_4-\gamma_4)\lambda^w_M rV^r_M+(1-\psi_4)\lambda^w_L rV^r_L}{(1-\theta_4)\lambda^w_H+(1-\mu_4-\gamma_4)\lambda^w_M+(1-\psi_4)\lambda^w_L}$. Since $\frac{x_M}{r} < V_{H_H}$ from $x_M < R^*_H$, k_0-type women have the incentive to reject low-type men. Therefore, a middle-type man always accepts a middle-type woman even if there are many H_0-type women who accept middle-type men.