

# FIXED POINT THEOREMS, RECURSIVE UTILITY AND THE SOLUTION TO THE BELLMAN EQUATION

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ABSTRACT. This study focuses on discrete-time models using recursive utility and considers the solution to a deterministic dynamic programming. Without topological assumptions on the state space of this problem, if the aggregator function composed of recursive utility functions is lower semicontinuous for the second argument and a lower boundary for the functions with certain properties exists, we show the existence of a fixed point of the Bellman operator in the restricted functional space and that this fixed point can be computed iteratively starting from the lower boundary. Under some topological assumptions, if an upper boundary for the functions with certain properties exists, we show the existence of a fixed point of the Bellman operator in the restricted functional space, that this fixed point is the value function, and that this fixed point can be computed by value iteration starting from the upper boundary.

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