

# Testing the Number of Components in Normal Mixture Regression Models

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October 30, 2014

## Abstract

Testing the number of components in finite normal mixture models is a long-standing challenge because of its non-regularity. This paper studies likelihood-based testing of the number of components in normal mixture regression models with heteroscedastic components. We construct a likelihood-based test of the null hypothesis of  $m_0$  components against the alternative hypothesis of  $m_0 + 1$  components for any  $m_0$ . The null asymptotic distribution of the proposed modified EM test statistic is the maximum of  $m_0$  random variables that can be easily simulated. The simulations show that the proposed test has very good finite sample size and power properties.

Key words: asymptotic distribution; modified EM test; likelihood ratio test; local MLE; normal mixture models; number of components

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