# Banks and Liquidity Crises in an Emerging Economy

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**Abstract**

This paper presents and analyzes a simple model where banking crises can occur when domestic banks are internationally illiquid. The model accounts for the basic features of banking crises after financial liberalization in emerging economies: (i) large capital inflow leads to high asset-prices volatility and (ii) enlarge the size of a banking crisis. The effects of some public policies are also examined.

## 1 Introduction

Financial crises have been cruel phenomena in many countries in many historical periods. In particular, after financial liberalizations across many parts of the world in the 1980s, crises have become more frequent and more costly events. Important examples include: Chile in 1982, Mexico in 1994, Argentina in 1995, Brazil in 1996, East Asia in 1997, and Russia in 1998. Some countries such as Latin America countries in the 1970s and 1980s experience crises because of inconsistent and unsustainable macroeconomic policies. In contrast, other countries such as East Asia in 1997 experience crises even when macroeconomic policies were consistent and sustainable. However, the empirical evidence strongly indicates that, in East Asia countries in 1990s after financial

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liberalizations, the short term external liabilities of its financial system were growing faster than its international reserves. That is, financial liberalization policies of these countries lead the financial systems to be *internationally illiquid* and add vulnerability to crises.

In addition, banking crises have often been accompanied by a sharp decline of asset prices historically. Some banks under strain demand liquidity and sell their assets to the market, which in turn causes a fall in asset prices and puts other banks under strain, forcing them to sell. A collapse in asset prices might cause a widespread financial crisis. Sarnoa and Taylor (1999) show that East Asian crisis of 1997 was precipitated by bursting asset prices, which had been fuelled by strong capital inflows.

This paper presents a simple banking model that can account for the observed effects of financial liberalization on crises. The model addresses the following basic characteristic of banking crises in emerging markets:

(i) Capital inflow increases the probability and size of a banking crisis.[1]

(ii) Financial institutions take on much short-term debt before a crisis occurs.[2]

(iii) Financial crises is closely linked to an asset-price boom and burst.[3]

Most of the literature has looked at combination of either (i) and (ii), or (ii) and (iii). The first class of papers includes Chang and Velasco (2000a, b, 2001), while the second class includes Allen and Gale (2004a, b). Considering the three ingredients are central to the analysis and contribution of this paper.

To analyze the effects of financial liberalization on banking crises, I extend the banking model developed by Chang and Velasco (2001) by incorporating the interbank asset markets. Chang and Velasco (2001) develop an open-economy version of the Diamond and Dybvig (1983) banking model.[4] They show that domestic bank runs, which is a panic

[1]See Kaminsky and Reinhart (1999), Reinhart and Reinhart (2009) and Reinhart and Rogoff (2008).

[2]See Sachs et al. (1996), Radelet and Sachs (1998), and Chang and Velasco (1998).

[3]See Kaminsky and Reinhart (1999) and Reinhart and Rogoff (2008).

[4]See also Chang and Velasco (2000a, b).

of domestic depositors, may interact with panics of international creditors. That is, banking runs may occur when domestic banks are internationally illiquid. Despite the elegance and usefulness of the model, it seem that the model has limitations in two ways. First, there is no aggregate uncertainty in the model: banking crises are "sunspot" phenomena. Some important empirical evidence are not on this side.[5] For example, Kaminsky and Reinhart (1999) study the relationship between banking crises and currency crises and find that banking crises typically precede currency crises which in turn exacerbates and deepens the baning crises, and these crises are related to weak economic fundamentals.[6] Second, there is no interbank asset markets in their model: trading assets between financial intermediaries, which play an important role under crises, are not modeled explicitly.

My analysis is also based on a banking model developed in Allen and Gale (1998, 2000, 2004a,b) and Allen et al. (2009). This paper extends their models into a small open economy. There are three periods in the usual way. Banks can borrow funds from domestic depositors and international creditors and hold one-period liquid international assets or two-period long term assets with a higher return. Banks face uncertain liquidity demands from their domestic depositors at the middle period. That is, there is the *aggregate uncertainty* that the overall level of the liquidity demands banks face is stochastic. Banks can meet the liquidity demands by using the liquid assets or selling the long term assets on a competitive interbank asset market where prices are endogenously determined by the demand and supply of liquidity in each state of nature.

I show that two types of equilibria can emerge depending on liquidity risk. When a probability of high liquidity risk is high, a no-default equilibrium exists, where all banks finds it optimal to keep enough international reserves and avoid defaults. When a probability of high liquidity risk is low, holding liquid reserves are costly, and the mixed equilibrium can emerge. In the mixed equilibrium, ex-ante identical banks take different portfolios. Some banks, called *risky banks*, invest heavily in the

[5]There is a long-standing debate whether banking crises are results of of self-fulfilling beliefs.

[6]See also Gorton (1988) and Calomiris and Gorton (1991).

long term asset and default in the bad state of nature when all consumers withdraw and a bank run occurs. As risky banks sell all their long term asset, in the bad state asset prices drop significantly and creditors obtain the liquidation proceeds instead of the promised repayments. The remaining banks, called *safe banks*, hold enough liquidity to always meet their commitments and buy the long term asset of the risky banks. The mixed equilibrium captures many features of a crisis in the emerging economies.

I then examine the effects of the two popular public policies, a liquidity requirement and public deposit insurance. The liquidity requirement is a constraint imposed on all banks holding a certain proportion of liquidity reserves, while under public deposit insurance policy depositors receive some goods when their banks go bankrupt. This paper show that crises can be eliminated at the expense of investment in higher yielding assets when the liquidity requirement is sufficiently restrictive. In contrast, public deposit insurance makes no contribution to stabilize the financial system because it encourages banks to take a risky portfolio.

There is an extensive literature on the implication of financial crises, financial intermediaries, and financial liberalization in open economies.

The first strand of literature focuses on the existence of multiple equilibria. In at least one equilibrium there is a banking panic while in another there is not. For example, Calvo (1988), Obstfeld (1996), Cole and Kehoe (1996, 2000), and Chang and Velasco (2000a, b, 2001). However, crises in their model were generated by sunspots and domestic asset market are not modeled explicitly.

The second is based on the business cycle view of crises. Allen and Gale (2000) develop a model of a banking crisis triggered by poor fundamentals and show that large movements in exchange rates are desirable to achieve optimal risk sharing. Although the model strategy is similar to mine, the liquidation value of the long-term asset is exogenous and the resulting equilibrium is typically symmetric in Allen and Gale (2000), while the value is determined endogenously in an asset market generating a mixed equilibrium in my framework.

The third strand of literature is on the implications of international financial frictions, capital flows and crises. Caballero and Krishnamurthy (2001) stress the interaction between domestic and international collateral for financial crises. Aoki, Benigno and Kiyotaki (2009), Mendoza and Quadrini (2010), and Mendoza (2010) develop an open economy version of a RBC model with collateral constraint to analyze the effects of financial liberalization on asset prices and the vulnerability of the financial systems. This paper differs in their approach to the volatility of asset prices after financial liberalization because in my model liquidity shortage can produce a fire-sale pricing in the market.

The paper proceeds as follows. Section 2 describes the model environment. The constrained efficient allocation is derived in Section 3. Two types of equilibria, a no-default equilibrium and a mixed equilibrium, are analyzed in Section 4. Section 5 examines the existence of equilibria while Section 6 presents numerical examples. The role of policies is analyzed in Section 7. Finally, Section 8 concludes.

# 2 The Model

I consider a small open economy with three periods indexed by $t = 0, 1, 2$. There is a single good at each period which can be used for consumption, investment or trade in an international market. For simplicity, the price of the good is fixed and normalized at one unit of international currency.[7]

The economy populated by a $[0, 1]$ continuum of ex ante identical domestic agents. Each agent has an endowment of one unit of the good only at period 0. The agents' time preferences are subject to a random shock at the beginning of period 1. With probability $\lambda$, an agent is an *early consumer* who only values consumption at period 1; with probability $1 - \lambda$, he is a *late consumer* who only values consumption at period 2. Type realizations are i.i.d. across agents and private information to that

[7] In developed countries, it is possible for domestic agents to borrow in the domestic currency and invest in foreign currency bonds. In contrast, in emerging countries foreign debt is usually denominated in foreign currency (dollars) rather than in domestic currency because foreign creditors fear inflation tax. The dollarized economy is essentially a real economy like described here.

agent. The ex-ante uncertainty about consumers' preferences generates a role for banks as liquidity providers as in Diamond and Dybvig (1983). The mass of banks is normalized to one, implying that a deposit market is competitive.

Let $c_1$ and $c_2$ denote the consumption levels of early and late consumers, respectively, and let $u(c)$ be their utility function. It is assumed that $u$ is strictly increasing, strictly concave, and twice continuously differentiable.

There is aggregate uncertainty about the fraction of early consumers in the model. Aggregate uncertainty is represented by a state of nature $\theta \in \{L, H\}$, and the probability of being an early consumer is given by

$$\lambda_\theta = \begin{cases} \lambda_L & \text{with prob. } \pi, \\ \lambda_H & \text{with prob. } 1-\pi, \end{cases}$$

where $\lambda_L < \lambda_H$ and $0 < \pi < 1$. For simplicity, $\lambda$ takes two values.

The economy is opened and small relative to the rest of the world in the sense that banks' behavior has no impact on the international prices. The rest of the world is risk neutral, and the gross return on a riskless asset is one. As in Chang and Velasco (2000, 2001), each bank can lend as much as he wants in the international market, while each bank can borrow at most an amount $f > 0$, where $f$ represents a country-level debt limit or a credit celling. The existence of the limit to foreign borrowing will be taken as exogenously given, but it is not too hard to justify. It may capture the idea that government regulations or lack of investor protection and monitoring to alleviate asymmetric information in emerging economies will limit credits from abroad. Then, $f$ can be interpreted as a degree of "financial liberalization" in the economy.

There are two types of domestic assets: a *short* asset and a *long* asset. Both are risk free. One unit of the good invested in the short asset at period $t$ yields one unit at period $t+1$ for $t = 0, 1$. One unit of the good invested in the long asset at period 0 yields $R > 1$ units at period 2. On average, emerging economies grow fast and has a high-return investment opportunity. The setting captures this feature by assuming $R > 1$. There is a trade-off between liquidity and returns: long-term investments have higher returns but take longer to mature.

There is a competitive domestic asset market at period 1.[8] Let $P$ denote the price of the long asset in terms of units of consumption at period 1. It is assumed that participation in this market is limited in the sense that only domestic banks can buy or sell the long asset. That is, the asset market is segmented, and foreign investors can not participate in the market because of government regulations or lack of knowledge about local properties. The assumption captures the dimension of financial underdevelopment in emerging economies. Since the bank can do anything that a consumer can do, there is no loss of generality in assuming that consumers deposit their entire endowment in a bank at period 0. After the realization of preference shocks, banks with low liquidity reserves will be able to sell their long asset and buy the short asset from banks with high liquidity reserves.

The timing of events is as follows. At period 0, banks take deposits from agents and borrow funds from the international market, and then divide these resources between international reserves, short and long assets. At period 1, the preference shocks are realized and the domestic asset market opens. At the end of period 1, domestic depositors and foreign investors who invest short term at period 0 receive payments from their banks. At period 2, domestic depositors who do not withdraw at period 1 and foreign investors who invest long term at period 0 and short term at period 1 withdraw and consume their consumption.

# 3 The Constrained Efficient Allocation

I shall begin with the constrained efficient allocation. The social planner treats agents symmetrically and make all the investment and consumption decisions in order to maximize the expected utility of a representative agent subject to a constraint of using a fixed payment at period 1 and international borrowing constraints. The planner's problem describes:

$$\max \; E_\theta[\lambda_\theta u(c_1) + (1-\lambda_\theta)u(c_{2\theta})],$$

[8]There are no markets for Arrow securities contingent on the future state at period 0.

subject to

$$x + y \leq 1 + b_{01} + b_{02}, \tag{1}$$

$$\lambda_\theta c_1 + b_{01} \leq y + b_{1\theta}, \tag{2}$$

$$(1 - \lambda_\theta)c_{2\theta} + b_{1\theta} + b_{02} \leq Rx + y + b_{1\theta} - b_{01} - \lambda_\theta c_1, \tag{3}$$

$$b_{01} + b_{02} \leq f, \tag{4}$$

$$b_{1\theta} + b_{02} \leq f, \tag{5}$$

$$c_1 \leq c_{2\theta}, \tag{6}$$

for any $\theta = L, H$ where $b_{01}$, $b_{02}$, and $b_{1\theta}$ are short term debt at period 0, long term debt at period 0, and short term debt at period 1 in state $\theta$ from the international market, respectively. The first is a resource constraint at period 0, which says that the investment in the short and long assets must be less or equal to the endowment plus short-and long-term international borrowings. Note that $y$ represents a composition of the short asset and international reserves because the gross return of both assets is the same one. The second constraint is the budget constraint at period 1 in state $\theta$, which says that the consumption at period 1 and the repayment to the international market must be less or equal to the amount of the short asset plus the short-term international borrowing. The third constraint is the budget constraint at period 2, which says that consumption at period 2 and repayment to the international market must be less or equal to the return from the long asset plus the amount of the short asset left over from period 1. The constraints (4) and (5) are the credit constraints at period 0 and 1, which say that total borrowing at any period in any states can not exceed the credit limit $f$. The final constraint is the incentive constraint, which says that the late consumers weakly prefer their own consumption to that of the early consumers for any states.

At the optimum, the borrowing constraint (4) and (5) in state $H$ will be binding:

$$b_{01} + b_{02} = f,$$
$$b_{1H} + b_{02} = f.$$

Otherwise, it would be possible to increase expected utility by borrowing more from abroad since $R > 1$. With the two equations, $b_{01} = b_{1H}$ is obtained easily. Similarly, at the optimum,

$$\lambda_H c_1 = y + b_{1H} - b_{01} = y.$$

If $\lambda_H c_1 + b_{01} < y + b_{1H}$, it would be possible to increase expected utility by holding $c_1$ constant and reducing $y$ since $R > 1$. Each bank holds an amount of the liquid asset just enough to satisfy the highest liquidity demand $\lambda_H c_1$ by early consumers in state $H$. This equation in turn leads to

$$(1 - \lambda_H)c_{2H} + b_{1H} + b_{02} = Rx,$$

or

$$(1 - \lambda_H)c_{2H} = Rx - f.$$

Thus the planner's problem is choose $y$ to maximize

$$\pi \left[ \lambda_L u \left( \frac{y}{\lambda_H} \right) + (1 - \lambda_L) u \left( \frac{R(1+f) - f - (R - 1 + \frac{\lambda_L}{\lambda_H})y}{1 - \lambda_L} \right) \right] + (1 - \pi) \left[ \lambda_H u \left( \frac{y}{\lambda_H} \right) + (1 - \lambda_H) u \left( \frac{R(1 + f - y) - f}{1 - \lambda_H} \right) \right].$$

This gives the first order condition that determines $y^*$:

$$\frac{\pi\lambda_L + (1-\pi)\lambda_H}{\lambda_H} u' \left( \frac{y}{\lambda_H} \right) - \pi \left( R - 1 + \frac{\lambda_L}{\lambda_H} \right) u' \left( \frac{R(1+f) - f - (R - 1 + \frac{\lambda_L}{\lambda_H})y}{1 - \lambda_L} \right) - R(1-\pi) u' \left( \frac{R(1+f-y) - f}{1 - \lambda_H} \right) = 0. \quad (7)$$

Differentiating a second time with respect to $y$ it can be seen that

$$\frac{\pi\lambda_L + (1-\pi)\lambda_H}{(\lambda_H)^2} u'' \left( \frac{y}{\lambda_H} \right) + \frac{\pi \left( R - 1 + \frac{\lambda_L}{\lambda_H} \right)^2}{1 - \lambda_L} u'' \left( \frac{R(1+f) - f - (R - 1 + \frac{\lambda_L}{\lambda_H})y}{1 - \lambda_L} \right) + \frac{R^2(1-\pi)}{1 - \lambda_H} u'' \left( \frac{R(1+f-y) - f}{1 - \lambda_H} \right) < 0$$

since $u'' < 0$. Thus the amount of the international liquidity reserves $y^*$ is determined uniquely. Then, it is easy to derive:

$$c_1^* = \frac{y^*}{\lambda_H}, \tag{8}$$

$$c_{2L}^* = \frac{R(1+f) - f - (R - 1 + \frac{\lambda_L}{\lambda_H})y^*}{1-\lambda_L}, \tag{9}$$

$$c_{2H}^* = \frac{R(1+f-y^*) - f}{1-\lambda_H}. \tag{10}$$

However, the optimal structure of the foreign debt $(b_{01}, \{b_{1\theta}\}_{\theta=L,H}, b_{02})$ is indeterminate. That is, any values of $b_{01} = b_{1H}$ and $b_{02}$ satisfying the binding (4) and (5) support a constrained efficient allocation.

# 4 Equilibrium

In what follows, I describe a decentralized economy in which banks offer demand deposits contracts to agents and trade assets through an interbank asset market. The model generates two types of equilibria, which will be discussed below. In one equilibrium, called a *no-default equilibrium*, runs do not occur, and all banks remain solvent. In the other equilibrium, called a *mixed equilibrium*, some banks experience a run and go bankrupt in state $H$, while others always remain solvent. A run occurs in the model only when the value of the bank's portfolio at period 2 does not suffice to repay at least $c_1$ to the late consumers. That is, self-fulfilling runs do not occur.

## 4.1 The no-default equilibrium

Consider first an equilibrium in which all banks offer identical *run-preventing contracts* initially and all depositors withdraw according to their time preferences. Because of competition among banks, they make their portfolio of $y$ in the short asset and international reserves and $x$ in the long asset at period 0 to maximize the expected utility of a representative agent. Like Allen and Gale (1998, 2004), it is assume that the deposit contract is incomplete in the sense that the repayment to both types of agents are not state contingent. The problem of banks is therefore:

$$\max\ E_\theta[\lambda_\theta u(c_1) + (1-\lambda_\theta)u(c_{2\theta})],$$

subject to

$$x + y \le 1 + b_{01} + b_{02}, \tag{11}$$
$$\lambda_\theta c_1 + b_{01} \le y + b_{1\theta}, \tag{12}$$
$$(1-\lambda_\theta)c_{2\theta} + b_{1\theta} + b_{02} \le R\left(x + \frac{y + b_{1\theta} - b_{01} - \lambda_\theta c_1}{P_\theta}\right), \tag{13}$$
$$b_{01} + b_{02} \le f, \tag{14}$$
$$b_{1\theta} + b_{02} \le f, \tag{15}$$
$$c_1 \le c_{2\theta}, \tag{16}$$

for any $\theta = L, H$. The constraints (11)–(13) are the resource constraints at period 0, 1 and 2 which have similar meaning in the planning problem. In state $\theta$, if $y + b_{1\theta} - b_{01} - \lambda_\theta c_1 > 0$, excess liquidity at period 1 can be used to buys $(y + b_{1\theta} - b_{01} - \lambda_\theta c_1)/P_\theta$ units of the long asset from other banks. If $y + b_{1\theta} - b_{01} - \lambda_\theta c_1 < 0$, the long asset held by bank must be sold in the market at period 1 to fund the shortfall of liquidity. The constraints (14) and (15) are the international credit constraints at period 0 and 1, and the final constraint is the incentive constraint. It is worth pointing out that the international interest rate the banks face is zero because they never default.

At the optimum, the borrowing constraint (14) will be binding:

$$b_{01} + b_{02} = f.$$

Otherwise, it would be possible to increase expected utility by borrowing more from abroad since $R > 1$. Similarly, when $R/P_\theta \ge 1$, it is optimal for the bank to borrow from abroad as much as possible and buy the long asset in state $\theta$ at period 1. Then,

$$b_{1\theta} + b_{02} = f,$$

for any $\theta = L, H$. With the three equations, $b_{01} = b_{1L} = b_{1H}$ is obtained easily.

The problem each bank solves at period 0 is to choose $c_1$ and $y$ to maximize

$$\pi \left[ \lambda_L u(c_1) + (1-\lambda_L) u \left( \frac{R(1+f-y+\frac{y-\lambda_L c_1}{P_L}) - f}{1-\lambda_L} \right) \right] \\ + (1-\pi) \left[ \lambda_H u(c_1) + (1-\lambda_H) u \left( \frac{R(1+f-y+\frac{y-\lambda_H c_1}{P_H}) - f}{1-\lambda_H} \right) \right],$$

subject to $c_1 > 0$ and $0 \leq y \leq 1+f$ taking prices $P_L$ and $P_H$ as given.

The first-order conditions for this with respect to the choice of $c_1$ and $y$ are

$$[\pi\lambda_L + (1-\pi)\lambda_H] u'(c_1) = \pi\lambda_L \frac{R}{P_L} u'(c_{2L}) + (1-\pi)\lambda_H \frac{R}{P_H} u'(c_{2H}), \quad (17)$$

$$\pi \left(1 - \frac{1}{P_L}\right) u'(c_{2L}) \leq (1-\pi) \left(\frac{1}{P_H} - 1\right) u'(c_{2H}), \quad (18)$$

with equality if $y < 1+f$, or equivalently $x > 0$.

Since bankruptcy can not occur in the equilibrium, market clearing requires that the aggregate demand for liquidity does not exceed the aggregate liquidity supply $y$:

$$\lambda_L c_1 < \lambda_H c_1 \leq y.$$

Since $\lambda_L < \lambda_H$, there is excess liquidity at period 1 in state $L$. In order for the interbank asset market to clear, it is necessary that:

$$P_L = R. \quad (19)$$

In this case, banks are willing to hold both the long asset and the excess liquidity between periods 1 and 2. If $P_L < R$, they will be willing to hold only the long asset while if $P_L > R$ they will be willing to hold only the liquid asset. Hence $P_L = R$ must hold.

Suppose that $\lambda_H c_1 < y$. Then, there is also excess liquidity at period 1 in state $H$, which implies that $P_H = R$. However, the long asset would dominate the short asset between period 0 and 1 and every bank invest only in the long asset, i.e., $y = 0$, a contradiction. Hence, in an equilibrium the following equation must hold:

$$\lambda_H c_1 = y. \tag{20}$$

Now substituting for $P_L$ and $c_1$ from (19) and (20) into (17) and (18) and arranging these equations yields

$$[\pi\lambda_L + (1-\pi)\lambda_H] u'\left(\frac{y}{\lambda_H}\right) = \pi\left[\lambda_H(R-1) + \lambda_L\right] u'\left(\frac{R(1+f) - f - (R - 1 + \frac{\lambda_L}{\lambda_H})y}{1-\lambda_L}\right) + \lambda_H R(1-\pi) u'\left(\frac{R(1+f-y)-f}{1-\lambda_H}\right), \tag{21}$$

which determines $y$ uniquely.

It is straightforward to see that equation (21) is equivalent to (7), which implies that the value of $y$ in the no-default equilibrium is the same as the value in the constrained efficient allocation. In addition, the values of $c_1$ and $c_{2\theta}$ derived here are also equivalent to (8)–(10). The next proposition states this result.

**Proposition 1** *The no-default equilibrium achieves the constrained efficient allocation.*

The price $P_H$ must ensure that banks are willing to hold both the liquid asset and the long asset between periods 0 and 1. Then the price $P_H$ can be derived from (18):

$$P_H = \frac{(1-\pi)Ru'(c_{2H})}{\pi(R-1)u'(c_{2L}) + (1-\pi)Ru'(c_{2H})} < 1. \tag{22}$$

Given $P_L = R > 1$, $P_H < 1$ must hold; otherwise the liquid asset is dominated by the long asset. The equilibrium prices defined by (19) and (22) fluctuate across states because of the inelasticity of liquidity supply at period 1. In addition, as in the planner's problem, the structure of the foreign debt $(b_{01}, \{b_{1\theta}\}_{\theta=L,H}, b_{02})$ is indeterminate because any values of $b_{01} = b_{1L} = b_{1H}$ and $b_{02}$ satisfying the binding international borrowing constraints can support the equilibrium.

Finally, for future reference, let $W^N$ denote the resulting expected utility in the no-default equilibrium.

## 4.2 The mixed equilibrium

Next, I characterize the mixed equilibrium where the price drops enough to generate bankrupt. As in the previous, holding excess liquidity is costly because doing so means forgoing the high return on the long asset.

First, one can show that in equilibrium, not all banks default simultaneously. Suppose that all banks make identical choices at period 0. If the fraction of early consumers are sufficiently high in state $H$, which violates the incentive constraint, all depositors try to withdraw their funds from their banks, and all banks try to sell the long asset for consumption goods at period 1. In this case, the price must be zero because no bank is willing to buy the long asset. However, this cannot be an equilibrium. Given a price of zero, a bank would be tempted to hold enough liquidity at period 0 and make a large capital gain by purchasing the long assets at period 1. Thus, an equilibrium where banks can default must be *mixed* in the sense that at least two types of banks, which take different strategies, exist.

Some banks hold a lot of the liquid asset at period 0 and offer deposit contracts promising low payments at period 1 to remain solvent. These banks are called *safe banks*. Other banks invest so heavily in the long asset and offer deposit contracts promising such high payments at period 1 that they may cause defaults. These banks are called *risky banks*. In state $L$, the safe banks have enough liquidity to meet depositors' liquidity demands and supply the remaining liquidity to the market. The risky banks can obtain the liquidity that they need to honor their repayment by selling the long asset. In state $H$, the risky banks will face high withdrawals and sell all of the long asset to meet the liquidity needs of their own customers. Since the liquidity supply is inelastic at period 1, the market is less liquid, and the liquidity shortage leads to a drop in price (so-called a *fire-sale price*), which forces the risky banks to go bankrupt. The safe banks will earn large capital gains because they hold enough liquidity in excess of their depositors' needs to enable them to buy the long asset at a fire-sale price.

Let us first consider the optimization problem of the safe banks. This is similar to the problem in the no-default equilibrium. Let $P_\theta$ denote the price of the asset market at period 1 in state $\theta \in \{L, H\}$. Given the market price $P_\theta$, the safe banks choose the consumption profile $(c_1^s, \{c_{2\theta}^s\}_{\theta \in \{L,H\}})$ to offer to depositors, the foreign debt profile $(b_{01}^s, b_{02}^s, \{b_{1\theta}^s\}_{\theta \in \{L,H\}})$ and the investment portfolio $(y^s, x^s)$ to maximize the expected utility of their customers. The problem of the safe banks is as follows:

$$\max \; E_\theta[\lambda_\theta u(c_1^s) + (1 - \lambda_\theta) u(c_{2\theta}^s)] \tag{23}$$

subject to

$$x^s + y^s = 1 + b_{01}^s + b_{02}^s, \tag{24}$$

$$\lambda_\theta c_1^s + b_{01}^s \leq y^s + b_{1\theta}^s, \tag{25}$$

$$(1 - \lambda_\theta) c_{2\theta}^s + b_{1\theta}^s + b_{02}^s \leq R \left( x^s + \frac{y^s + b_{1\theta}^s - b_{01}^s - \lambda_\theta c_1^s}{P_\theta} \right), \tag{26}$$

$$b_{01}^s + b_{02}^s \leq f, \tag{27}$$

$$b_{1\theta}^s + b_{02}^s \leq f, \tag{28}$$

$$c_1^s \leq c_{2\theta}^s, \tag{29}$$

for any $\theta = L, H$. The constraints (24)–(25) are the resource constraints at period 0, 1 and 2 which have similar meaning in the no default equilibrium. In state $\theta$, the safe bank has $y^s + b_{1\theta}^s - b_{01}^s - \lambda_\theta c_1^s$ units of excess liquidity and buys $(y^s + b_{1\theta}^s - b_{01}^s - \lambda_\theta c_1^s)/P_\theta$ units of the long asset from the risky banks. The constraints (27) and (28) are the credit constraints at period 0 and 1, and the final constraint is the incentive constraint. Because the safe banks never default, the international interest rate they face is zero.

As in the no default equilibrium, the international borrowing constraints (27) and (28) will be binding at the optimum:

$$b_{01}^s + b_{02}^s = f, \tag{30}$$

$$b_{1\theta}^s + b_{02}^s = f, \quad \theta = L, H. \tag{31}$$

If $b_{01}^s + b_{02}^s < f$, it would be possible to increase expected utility by increasing $b_{02}^s$ since $R > 1$. Similarly, if $b_{1\theta}^s + b_{02}^s < f$, it would be possible to increase expected utility by increasing $b_{1\theta}^s$ and buying the long term asset at period 1 since $R/P_\theta > 1$ for any $\theta = L, H$. With the three constraints, $b^s_{01} = b^s_{1L} = b^s_{1H}$ is obtained, and the resource constraints, (24) and (26) can be rewritten as:

$$x^s + y^s = 1 + f, \tag{32}$$

$$(1-\lambda_\theta)c^s_{2\theta} + f = R\left(x^s + \frac{y^s - \lambda_\theta c^s_1}{P_\theta}\right), \tag{33}$$

for any $\theta = L, H$.

The first-order conditions for the problem are:

$$[\pi\lambda_L + (1-\pi)\lambda_H]u'(c^s_1) = \pi\lambda_L \frac{R}{P_L} u'(c^s_{2L}) + (1-\pi)\lambda_H \frac{R}{P_H} u'(c^s_{2H}), \tag{34}$$

$$\pi\left(1 - \frac{1}{P_L}\right)u'(c^s_{2L}) \leq (1-\pi)\left(\frac{1}{P_H} - 1\right)u'(c^s_{2H}), \tag{35}$$

with equality if $x^s > 0$. Given the asset price $P_\theta$ at period 1, the vector $\{(c^s_1, \{c^s_{2\theta}\}_{\theta\in\{L,H\}}), (y^s, x^s)\}$ is determined by the conditions (32)–(35). Note that the structure of the foreign debt $(b^s_{01}, b^s_{02}, \{b^s_{1\theta}\}_{\theta\in\{L,H\}})$ is indeterminate because any values of $b^s_{01}, \{b^s_{1\theta}\}_{\theta\in\{L,H\}}$, and $b^s_{02}$ satisfying (30) and (31) support an equilibrium.

Consider next the optimization problem of the risky banks. In state $L$, they can offer high repayments to depositors and can borrow funds from the international market at period 1. In state $H$, they sell all of the long asset to meet the liquidity needs of their depositors and go bankrupt. In particular, the international creditors reject new lending to the risky banks at period 1 (i.e., $b^r_{1H} = 0$) and try to withdraw their funds from them in the same way as domestic depositors. Since the risky banks may fail to meet their obligation, the international interest rate they face will be greater than zero. Let $r_1$ and $r_2$ denote the short-and long-term interest rate they face in the international market, respectively. Given the market price $P_\theta$ and the interest rates $r_1$ and $r_2$, the risky banks choose the consumption profile $(c^r_1, c^r_{2L})$ to offer to depositors, the foreign debt profile $(b^r_{01}, b^r_{02}, b^r_{1L})$ and the investment portfolio $(y^r, x^r)$ to maximize the expected utility. The problem of the risky banks is as follows:

$$\max\ \pi[\lambda_L u(c^r_1) + (1-\lambda_L)u(c^r_{2L})] + (1-\pi)u\left(\frac{c^r_1}{c^r_1 + (1+r_1)b^r_{01}}(y^r + P_H x^r)\right) \tag{36}$$

subject to

$$x^r + y^r = 1 + b^r_{01} + b^r_{02}, \tag{37}$$

$$\lambda_L c^r_1 + (1 + r_1)b^r_{01} \leq y^r + b^r_{1L} + P_L x^r, \tag{38}$$

$$(1 - \lambda_L)c^r_{2L} + b^r_{1L} + (1 + r_2)b^r_{02} \leq R\left(x^r - \frac{(1 + r_1)b^r_{01} + \lambda_L c^r_1 - y^r - b^r_{1L}}{P_L}\right), \tag{39}$$

$$b^r_{01} + b^r_{02} \leq f, \tag{40}$$

$$b^r_{1L} + b^r_{02} \leq f, \tag{41}$$

$$c^r_1 \leq c^r_{2L}. \tag{42}$$

Note that the risky banks liquidate all of their assets and distribute its in proportion to the creditors' claims at period 1 in state $H$. Each domestic depositor receives a fraction $c^r_1/(c^r_1 + (1 + r_1)b^r_{01})$ of the asset value and international creditors receive the rest. Note also that the whole long term debt is defaulted on in state $H$. The constraints (37)–(39) are the resource constraints at period 0, 1 and 2. The risky banks demand $(1 + r_1)b^r_{01} + \lambda_L c^r_1 - y^r - b^r_{1L}$ units of good and sell $((1 + r_1)b^r_{01} + \lambda_L c^r_1 - y^r - b^r_{1L})/P_L$ units of the long asset to the safe banks in state $L$, while they sell all of the long asset and demand $P_H x^r$ units of good in state $H$. The constraints (40) and (41) are the international credit constraint at period 1 and 2, while the constraint (42) is the incentive constraint in state $L$.

Solutions of the risky banks are:

$$y^r + x^r = 1 + f \tag{43}$$

$$b^r_{01} + b^r_{02} = f \tag{44}$$

$$b^r_{1L} + b^r_{02} = f \tag{45}$$

$$u'(c^r_1) + \frac{1 - \pi}{\pi\lambda_L}u'\left(\frac{c^r_1(y^r + P_H x^r)}{c^r_1 + (1 + r_1)b^r_{01}}\right)\frac{(y^r + P_H x^r)(1 + r_1)b^r_{01}}{(c^r_1 + (1 + r_1)b^r_{01})^2} = \frac{R}{P_L}u'(c^r_{2L}) \tag{46}$$

$$\pi R\left(1 - \frac{1}{P_L}\right)u'(c^r_{2L}) = (1 - \pi)u'\left(\frac{c^r_1(y^r + P_H x^r)}{c^r_1 + (1 + r_1)b^r_{01}}\right)\frac{c^r_1(1 - P_H)}{c^r_1 + (1 + r_1)b^r_{01}} + \mu_8, \tag{47}$$

$$\pi \left( r_2 - \frac{R}{P_L} r_1 \right) u'(c^r_{2L}) = (1-\pi) u' \left( \frac{c^r_1 (y^r + P_H x^r)}{c^r_1 + (1+r_1) b^r_{01}} \right) \frac{(y^r + P_H x^r)(1+r_1) c^r_1}{(c^r_1 + (1+r_1) b^r_{01})^2} - \mu_5 - \mu_6 + \mu_7, \quad (48)$$

where $\mu_5$, $\mu_6$, $\mu_7$, $\mu_8$ are the Lagrange multipliers on the non-negativity constraints for $b^r_{1L}$, $b^r_{01}$, $b^r_{02}$, and $y^r$, respectively.

In equilibrium, domestic depositors must be indifferent between depositing their funs in a safe or risky bank; otherwise, one type of banks will attract no depositors. Let $W^s$ and $W^r$ denote the expected utility of safe and risky banks, respectively. The two expected utilities must be equalized as follows:

$$W^s = W^r. \quad (49)$$

The asset market at period 1 must clear in both states. Let $\rho$ and $1-\rho$ denote a proportion of the safe banks and the risky banks, respectively. In state $L$, the risky banks demand liquidity $(1-\rho)((1+r_1)b^r_{01}+\lambda_L c^r_1 - y^r - b^r_{1L})$, and the safe banks supply their excess liquidity $\rho(y^s+b^s_{1L}-\lambda_L c^s_1 - b^s_{01})$ for the long asset. Market clearing requires the demand for and the supply of liquidity to be equal at price $P_L$ as follows:

$$\rho(y^s + b^s_{1L} - \lambda_L c^s_1 - b^s_{01}) = (1-\rho)((1+r_1) b^r_{01} + \lambda_L c^r_1 - y^r - b^r_{1L}). \quad (50)$$

In state $H$, the risky banks sell all of their long asset $(1-\rho)x^r$ at price $P_H$, and the safe banks supply their excess liquidity $\rho(y^s + b^s_{1H} - \lambda_H c^s_1 - b^s_{01})$ for the long asset. The market clearing requires:

$$\rho(y^s + b^s_{1H} - \lambda_H c^s_1 - b^s_{01}) = (1-\rho) P_H x^r. \quad (51)$$

In state $H$, the short term debt is partially repudiated, while the long term debt is not. The international creditors who make short-term lending at period 0 receive $(1+r_1)b^r_{01}$ in state $L$ and $(1+r_1)b^r_{01}(y^r+P_H x^r)/(c^r_1 + (1+r_1)b^r_{01})$ in state $H$ at period 1, while the international creditors who make long-term lending receive $(1+r_2)b^r_{01}$ in state $L$ and nothing in state $H$ at period 2. Then, the no-arbitrage conditions are:

$$1 = \pi(1+r_1) + (1-\pi) \frac{(1+r_1)(y^r + P_H x^r)}{c^r_1 + (1+r_1) b^r_{01}}, \quad (52)$$

$$1 = \pi(1+r_2). \quad (53)$$

As in Chang and Velasco (2000), a term structure of interest rates emerges endogenously. That is, the long term debt is more expensive than the short term debt ($r_1 < r_2$), which is often relevant empirically.

The mixed equilibrium is characterized by the vector $(c_1^s, \{c_{2\theta}^s\}, \{b_{0t}^s\}, \{b_{1\theta}^s\}, y^s, c_1^r, c_{2L}^r, \{b_{0t}^r\}, \{b_{1\theta}^r\}, y^r, \{P_\theta\}, \{r_t\}, \rho)$ satisfying (32)–(35), (39), (43)–(53).

# 5 Existence of Equilibria

In the previous section, I characterized the two types of equilibria. In this section, I analyze the parameter space in which they exist. The key element for the existence of the equilibria is whether the strategies of the risky banks are optimal. The no-default equilibrium exists if no bank finds it optimal to default given the prices $P_L$ and $P_H$ satisfying (19) and (22). On the other hand, the mixed equilibrium exists if some banks prefer portfolio allocations and deposit contracts that support default. That is, in order for the possibility of bank runs to arise in equilibrium, some banks must choose risky portfolios, and the remaining banks then choose safe portfolios.

Let us consider the problem of a bank that tries to choose a risky portfolio in a situation where all banks take a safe portfolio. The problem is quite similar to that of the risky banks in the mixed equilibrium. The difference is that the bank takes the market prices $P_L = R$ and $P_H$ defined by (22) as given. The deviating bank chooses the consumption profile $(c_1^d, c_{2L}^d)$, the portfolio $(y^d, x^d)$, and the foreign debt profile $(b_{01}^d, b_{02}^d, b_{1L}^d, b_{1H}^d)$ in order to maximize the following expected utility:

$$\max \ \pi[\lambda_L u(c_1^d) + (1-\lambda_L)u(c_{2L}^d)] + (1-\pi)u\left(\frac{c_1^d}{c_1^d + (1+r_1)b_{01}^d}(y^d + P_H x^d)\right) \tag{54}$$

subject to

$$x^d + y^d = 1 + b_{01}^d + b_{02}^d, \tag{55}$$

$$\lambda_L c_1^d + (1 + r_1) b_{01}^d \leq y^d + b_{1L}^d + P_L x^d, \tag{56}$$

$$(1 - \lambda_L) c_{2L}^d + b_{1L}^d + (1 + r_2) b_{02}^d \leq R \left( x^d - \frac{(1 + r_1) b_{01}^d + \lambda_L c_1^d - y^d - b_{1L}^d}{P_L} \right), \tag{57}$$

$$b_{01}^d + b_{02}^d \leq f, \tag{58}$$

$$b_{1L}^d + b_{02}^d \leq f, \tag{59}$$

$$c_1^d \leq c_{2L}^d. \tag{60}$$

Since the bank defaults in state $H$, $b_{1H}^d = 0$ again. These constraints have similar meanings to that of the maximization problem of the risky banks in the mixed equilibrium. The solutions for the problem are given by

$$y^d + x^d = 1 + f, \tag{61}$$

$$b_{01}^d = b_{1L}^d = b_{1H}^d = 0, \quad b_{02}^d = f, \tag{62}$$

$$c_1^d = c_{2L}^d = y^d + R x^d - \frac{f}{\pi}, \tag{63}$$

$$(1 - \pi)(1 - P_H) u' \left( y^d + P_H x^d \right) \leq \pi (R - 1) u'(c_{2L}^d), \tag{64}$$

with equality if $y^d > 0$ where $P_H$ is given by (22).

A deviating bank borrows long-term from the international market as much as possible and invests a large proportion of its resources into the long asset. Then, it can provide good liquidity insurance and returns to depositors in state $L$ but default in state $H$.

Let $W^d$ denote the maximized expected utility corresponding to the solutions:

$$W^d = \pi u \left( y^d + R x^d - \frac{f}{\pi} \right) + (1 - \pi) u \left( y^d + P_H x^d \right),$$

where $P_H$ is given by (22) and $(y^d, x^d)$ solves (61) and (64). The condition $W^d \geq W^N$ ensures that a bank has an incentive to chooses a risky portfolio in a situation where all banks take a safe portfolio. That is, the condition $W^d \geq W^N$ is necessary for the existence of the mixed equilibrium. The next proposition summarizes the existence of the equilibria.

**Proposition 2** *If $W^N > W^d$, then there exists a no-default equilibrium.*

It will be optimal for banks to avoid default in state $H$ when the probability of state $H$ occurring is high and the risk aversion of agents is high.

# 6 Examples

I illustrate the equilibria described above with the numerical examples.

## 6.1 Basic Examples

First, I assume that depositors have the log utility function:

$$u(c) = \log(c).$$

The liquidity shocks and the return on the long asset are assumed to be:

$$\lambda_L = 0.8, \quad \lambda_H = 0.81, \quad \text{and} \quad R = 1.5.$$

In Example 1, I fix the probability of state $L$, $\pi = 0.6$, and successively increase the credit celling, $f$: 0.3 in Example 1A, 0.5 in Example 1B, and 0.7 in Example 1C. In Example 2, I fix the probability, $\pi = 0.8$, and successively increase $f$: 0.3 in Example 2A, 0.5 in Example 2B, and 0.7 in Example 2C. These parameter values produce results that are typical of other simulations.

Table 1 shows the types of equilibria, the volatility of the asset prices $P_L/P_H$, the proportion of the safe bank $\rho$, and expected utility $E[u]$ , for various values of $\pi$ and $f$.

In Example 1, since every bank does not have an incentive to choose a risky portfolio, there exists a unique no-default equilibrium.[9] In the no-default equilibrium, every bank has enough liquidity reserves to cover high liquidity demands. The equilibrium asset prices fluctuate across

[9] The expected utility that a deviating bank can offer $W^d$ is 0.1634 in Example 1A, 0.2067 in Example 1B, and 0.2424 in Example 1C, which are less than the respective expected utility in the no-default equilibrium.

| Ex. | $\pi$ | $f$ | Types of eqm. | Price volatility ($P_L/P_H$) | $\rho$ | $E[u]$ |
|---|---|---|---|---|---|---|
| 1A | 0.6 | 0.3 | No default | 1.5000/0.6628=2.2631 | 1.0000 | 0.1728 |
| 1B | 0.6 | 0.5 | No default | 1.5000/0.6628=2.2631 | 1.0000 | 0.2316 |
| 1C | 0.6 | 0.7 | No default | 1.5000/0.6628=2.2631 | 1.0000 | 0.2872 |
| 2A | 0.8 | 0.3 | Mixed | 1.2620/0.5189=2.4321 | 0.9723 | 0.1741 |
| 2B | 0.8 | 0.5 | Mixed | 1.2947/0.4905=2.6396 | 0.9707 | 0.2328 |
| 2C | 0.8 | 0.7 | Mixed | 1.3296/0.4646=2.8618 | 0.9700 | 0.2883 |

Table 1: Numerical Examples

states but relaxing the credit ceiling, which is interpreted as financial liberalization, has no impacts on the prices and its volatility. Table 2 gives the allocations in the no-default equilibrium. As the credit ceiling, $f$, increases, the amount of reserves, the long term investment, and payments to depositors increase, resulting in improving the expected utility. Note that early consumers receive the same level of consumption, $c_1$, in each state while the late consumers have different levels of consumption in each state because they receive the residual value of the portfolio and this depends on the asset price, $P_\theta$. Since there are no banking defaults in equilibrium, every bank faces the low interest rates in the international capital market (i.e., $r_1 = r_2 = 0$), which leads to the indeterminacy of the foreign debt structure. Note that these allocations are constrained efficient as stated in Proposition 1.

| Ex. | $\pi$ | $f$ | $(y, x)$ | $(c_1, c_{2L}, c_{2H})$ | $(b_{01}, b_{1L}, b_{1H}, b_{02})$ |
|---|---|---|---|---|---|
| 1A | 0.6 | 0.3 | (0.8888, 0.4112) | (1.0973, 1.6389, 1.6674) | indeterminate |
| 1B | 0.6 | 0.5 | (0.9427, 0.5573) | (1.1638, 1.7379, 1.7682) | indeterminate |
| 1C | 0.6 | 0.7 | (0.9966, 0.7034) | (1.2304, 1.8370, 1.8689) | indeterminate |

Table 2: Allocations in the no-default equilibrium

Under the parameters in Example 2, a bank has an incentive to choose a risky portfolio when all banks take a safe portfolio because the probability of state $H$, $1 - \pi$, is sufficiently low. That is, the no-default equilibrium no longer exists, and there exists a mixed equilibrium.[10] Ta-

[10]The expected utility that a deviating bank can offer $W^d$ is 0.2444 in Example

ble 1 shows that in contrast to the no-default equilibrium capital inflow has significant impacts on the prices and its volatility. Capital inflow increases the asset price in state $L$, $P_L$ and decreases the price in state $H$, $P_H$, resulting in increase in the price volatility. In addition, large capital inflow increases the proportion of risky banks, $1-\rho$, which means that capital inflow increases the size of a banking crisis. However, as capital inflow increases, the expected utility increases because the level of consumption are improved.

Table 3 gives the allocations in the mixed equilibrium. The safe banks hold large amounts of liquid international reserves and offer deposit contracts promising low payments at period 1. The risky banks invest only in the long term asset and offer deposit contracts promising high payments at period 1. The risky banks borrow long term from the international creditors up to their credit limit while the safe banks are indifferent between the structure of the foreign debt in every example. When liquidity demands are low ($\theta = L$), the safe banks have excess liquidity which they supply to the market by buying the long asset. The risky banks obtain the liquidity they need to honor their deposit contracts by selling the long asset. When liquidity demands are high ($\theta = H$), the market for the long asset is less liquid because the safe banks must devote more of their liquidity to satisfying the needs of their own customers. This liquidity shortage leads to a drop in the price of the long asset, which forces the risky banks to go bankrupt and liquidate their entire stocks of the long asset. The increase in the supply of the long asset can lead to a sharp drop in prices. In this case there is "cash-in-the-market" pricing. The safe banks hold just enough liquidity in excess of their customers' needs to enable them to buy up the long asset at a firesale price. The low price compensates them for the cost of holding the extra liquidity when liquidity demands are low and prices are high.

In addition, in the mixed equilibrium the ratio of short term debt to international liquidity reserves can be increasing in capital inflow Let us

2A, 0.2980 in Example 2B, and 0.3473 in Example 2C, which are larger than the respective expected utility in the no-default equilibrium.

| Ex. | $\pi$ | $f$ | $(y^s, x^s)$<br>$(y^r, x^r)$ | $(c_1^s, c_{2L}^s, c_{2H}^s)$<br>$(c_1^r, c_{2L}^r, y^r + P_H x^r)$ | $(b_{01}^s, b_{1L}^s, b_{1H}^s, b_{02}^s)$<br>$(b_{01}^r, b_{1L}^r, b_{1H}^r, b_{02}^r)$ |
|---|---|---|---|---|---|
| 2A | 0.8 | 0.3 | (0.9085, 0.3915) | (1.0979, 1.6155, 1.8040) | indeterminate |
| | | | (0.0000, 1.3000) | (1.3251, 1.5750, 0.6746) | (0.0000, 0.0000, 0.0000, 0.3000) |
| 2B | 0.8 | 0.5 | (0.9652, 0.5348) | (1.1643, 1.7068, 1.9473) | indeterminate |
| | | | (0.0000, 1.5000) | (1.4026, 1.6250, 0.7358) | (0.0000, 0.0000, 0.0000, 0.5000) |
| 2C | 0.8 | 0.7 | (1.0212, 0.6788) | (1.2306, 1.7984, 2.0899) | indeterminate |
| | | | (0.0000, 1.7000) | (1.4848, 1.6750, 0.7898) | (0.0000, 0.0000, 0.0000, 0.7000) |

Table 3: Allocations in the mixed equilibrium

define the ratio as

$$\eta \equiv \frac{\rho(\bar{\lambda} c_1^s + b_{01}^s) + (1-\rho)(\bar{\lambda} c_1^r + (1+r_1) b_{01}^r)}{\rho y^s + (1-\rho) y^r},$$

where $\bar{\lambda} \equiv \pi \lambda_L + (1-\pi)\lambda_H$ is the average fraction of early consumers. The term $\rho(\bar{\lambda} c_1^s + b_{01}^s) + (1-\rho)(\bar{\lambda} c_1^r + (1+r_1) b_{01}^r)$ represents the average total short liabilities of the banking system at period 1, while the term $\rho y^s + (1-\rho) y^r$ is the total international liquidity reserves of the system at the same period. A ratio higher than one implies that international reserves would have not been sufficient to repay maturing debt, capturing a difficult liquidity situation of an economy. In whole examples, $b_{01}^r = 0$ holds while $b_{01}^s$ is indeterminate and takes a value in $[0, f]$. Suppose that $b_{01}^s = \nu f$ (or equivalently, $b_{02}^s = (1-\nu) f$) where $\nu = 0.2$. In the case of Example 2, then the ratio $\eta$ is increasing in $f$ and takes the values 1.0686 when $f = 0.3$; 1.1062 when $f = 0.5$; and 1.1396 when $f = 0.7$, which is consistent with empirical evidence by Chang and Velasco (1998) and Radelet and Sachs (1998).

The model presented here captures the basic features of the financial crises in emerging economies. It is then natural to ask whether the expected utility in the mixed equilibrium is larger than the one of a constrained efficient allocation. The examples provide a somewhat surprising result. The expected utility provided to agents by the planner are 0.1729 in Example 2A, 0.2318 in Example 2B, and 0.2873 in Example 2C, which are less than the values in the mixed equilibrium. The intuition for this result is related to contingency of banking contracts. Under the assumption that banks are restricted to using non-contingent banking contracts, the choices of the planner and banks are distorted. As we have seen, some banks can not meet their commitments and go bankrupt in one state under incomplete contracts. In this case, depositors receive only the liquidated value of their banks' portfolio rather than the promised payment. However, this fact means that default relaxes the constraint of incomplete contracts and allows the banks to offer the deposit contract more contingent on the state of nature, resulting in more efficient risk sharing. This result implies that there is no justification for government interventions that prevent a financial crisis.[11]

## 6.2 Risk Aversion

So far I have assumed that depositors have a log utility function. This implies a constant relative risk aversion equal to one. I now assume that depositors have a constant relative risk aversion utility function (CRRA) given by

$$u(c) = \frac{c^{1-\sigma}}{1-\sigma},$$

where $\sigma \geq 1$ represents the degree of risk aversion. Table 4 illustrates the equilibrium values corresponding to $\sigma = 1$, 2 and 3. Other parameters take the same values ($\lambda_L = 0.8$, $\lambda_H = 0.81$, and $R = 1.5$) as in Example 2.

Note that as a relative risk aversion increases depositors require banks to hold more liquidity reserves and to provide better insurance against liquidity shocks. Then, a higher relative risk aversion reduces the profit of a deviating bank from the no-default equilibrium, which means that it enlarges the range of parameter for which the no-default equilibrium exists. When a relative risk aversion is sufficiently close to one ($\sigma = 2$), the mixed equilibrium still exists. In this equilibrium, the risky banks also hold some liquidity reserves between period 0 and 1 and reduce the amount of the long asset they sell at period 1. Then the asset market becomes more liquid, and the price of the long asset in state $L$ is increased,

[11] Allen and Gale (2004) provide a more detailed discussion.

| Ex. | $\sigma$ | $\pi$ | $f$ | Foreign debt | Price volatility $(P_L/P_H)$ | $\rho$ | $E[u]$ |
|---|---|---|---|---|---|---|---|
| 2A | 1 | 0.8 | 0.3 | Mixed | 1.2620/0.5189=2.4321 | 0.9723 | 0.1741 |
| 2B | 1 | 0.8 | 0.5 | Mixed | 1.2947/0.4905=2.6396 | 0.9707 | 0.2328 |
| 2C | 1 | 0.8 | 0.7 | Mixed | 1.3296/0.4646=2.8618 | 0.9700 | 0.2883 |
| 3A | 2 | 0.8 | 0.3 | Mixed | 1.5000/0.3952=3.7955 | 0.9888 | -0.8465 |
| 3B | 2 | 0.8 | 0.5 | Mixed | 1.5000/0.4012=3.7388 | 0.9918 | -0.7981 |
| 3C | 2 | 0.8 | 0.7 | Mixed | 1.5000/0.4049=3.7046 | 0.9936 | -0.7550 |
| 4A | 3 | 0.8 | 0.3 | No default | 1.5000/0.4237=3.5402 | 1.0000 | -0.3598 |
| 4B | 3 | 0.8 | 0.5 | No default | 1.5000/0.4237=3.5402 | 1.0000 | -0.3198 |
| 4C | 3 | 0.8 | 0.7 | No default | 1.5000/0.4237=3.5402 | 1.0000 | -0.2862 |

Table 4: Numerical Examples for $u(c) = c^{1-\sigma}/(1-\sigma)$ where $\sigma > 1$.

resulting in significant asset-price volatility. When a relative risk aversion is sufficiently high ($\sigma = 3$), the equilibrium is default-free because no bank has an incentive to take a risky portfolio.[12]

# 7 Policy Implications

In this section, I examine some policy implications. As stated earlier, banking default restore the contingency of deposit contracts, and the public policy can not improves welfare. However, a government would want to put much value on the financial stability rather than ex ante efficiency because a crisis may have significant negative impacts on the real sector (e.g., increasing unemployment, decreasing output, etc.), which are not modeled here. Can the government eliminate a crisis at the expense of welfare? To answer this question, I consider two popular policies that are often implemented in practice: a liquidity requirement and a government deposit insurance. To gain some insight into the complex effects of the policy interventions, I employ Example 2 presented in the previous section.

[12]The basic equilibrium properties are the same in Example 1.

## 7.1 Liquidity Requirement

The liquidity requirement is a constraint imposed on all banks holding a certain proportion of liquidity reserves in the bank's portfolio.[13] Specifically, the government forces banks to invest at least the fraction $\xi$ of their available resources $(1 + f)$ in the liquidity reserves at period 0 as:

$$y \geq \xi(1 + f),$$

where $0 \leq \xi \leq 1$. Since this constraint reduces a profit of a bank deviating from the no-default equilibrium, it enlarges the range of parameter for which the no-default equilibrium exists. This policy may be irrelevant to the safe banks because they already hold enough international liquid reserves. Table 5 compares the mixed equilibrium with liquidity requirements, $\xi = 0$, 0.2 and 0.5. Table 5 shows that the policy eliminates the possibility of a crisis at the expense of the expected utility as long as it is sufficiently severe ($\xi \geq 0.5$). However, if the liquidity constraint is not restrictive ($\xi = 0.2$), the mixed equilibrium still exists under the constraint. In the equilibrium, the size of banking defaults, the volatility of the asset market, and welfare loss are increased. These examples imply that a tepid liquidity requirement could harm rather than stabilize the banking system.

| Ex. | $\pi$ | $f$ | $\xi$ | Types of eqm. | Price volatility ($P_L/P_H$) | $\rho$ | $E[u]$ |
|---|---|---|---|---|---|---|---|
| 2A | 0.8 | 0.3 | 0 | Mixed | 1.2620/0.5189=2.4321 | 0.9723 | 0.1741 |
| 2B | 0.8 | 0.5 | 0 | Mixed | 1.2947/0.4905=2.6396 | 0.9707 | 0.2328 |
| 2C | 0.8 | 0.7 | 0 | Mixed | 1.3296/0.4646=2.8618 | 0.9700 | 0.2883 |
| 5A | 0.8 | 0.3 | 0.2 | Mixed | 1.3114/0.4802=2.7309 | 0.9619 | 0.1739 |
| 5B | 0.8 | 0.5 | 0.2 | Mixed | 1.3265/0.4371=3.0348 | 0.9600 | 0.2325 |
| 5C | 0.8 | 0.7 | 0.2 | Mixed | 1.4526/0.4007=3.6252 | 0.9593 | 0.2879 |
| 6A | 0.8 | 0.3 | 0.5 | No default | 1.5000/0.4243=3.5352 | 1.0000 | 0.1729 |
| 6B | 0.8 | 0.5 | 0.5 | No default | 1.5000/0.4243=3.5352 | 1.0000 | 0.2318 |
| 6C | 0.8 | 0.7 | 0.5 | No default | 1.5000/0.4243=3.5352 | 1.0000 | 0.2873 |

Table 5: The effects of liquidity regulations: $y \geq \xi(1 + f)$

[13] The liquidity coverage ratio is envisioned by Basel III.

## 7.2 Government Deposit Insurance

I next consider the public deposit insurance provided by the government and financed by taxes.[14] The importance of insuring depositors in the event of a run is generally acknowledged because it reduces their incentive to withdraw their funds early.[15] Under the deposit insurance policy, depositors receive some funds from the government when their banks go bankrupt. Specifically, I assume that in state $H$ at period 1 the government imposes a lump-sum tax on the safe banks, $\tau$ and transfer $\phi$ to the depositors of the risky banks. Then the budget constraint of the safe banks at period 1 in state $H$ is modified as:

$$\lambda_H c_1^s + b_{01}^s + \tau \leq y^s + b_{1H}^s,$$

while in state $H$ all depositors of the risky banks receive:

$$\frac{c_1^r(y^r + P_H x^r)}{c_1^r + (1 + r_1)b_{01}^r} + \phi.$$

Therefore, the government resource constraint is given by:

$$(1 - \rho)\phi = \rho\tau.$$

Note that since the deposit insurance policy benefits a deviating bank it enlarges the range of parameter for which the mixed equilibrium exists.

Table 6 shows the equilibrium values with and without the public deposit insurance. In these examples, the government set the transfer level ($\phi$ = 0, 0.1, and 0.2) at period 0, which means that the tax level $\tau$ is determined endogenously by the government budget constraint . Table 6 shows that the public deposit insurance stabilizes the asset price volatility but increases the size of defaults and decreases welfare because more banks choose a risky portfolio. Interestingly, the effect of increasing capital inflow on the number of risky banks is very different. In the model, the public deposit insurance increases the fragility of the banking systems, which is consistent with the empirical evidence by Demirguc-Kunt and Detragiache (2002).

[14]Most emerging countries of Latin America and East Asia had not established the public deposit insurance before a crisis. See Demirguc-Kunt and Detragiache (2002).

[15]In the classic work of Diamond and Dybvig (1983), deposit insurance is an optimal policy when banking stability is threatened by self-fulfilling depositor runs.

| Ex. | $\pi$ | $f$ | DI ($\phi$, $\tau$) | Types of eqm. | Price volatility ($P_L/P_H$) | $\rho$ | $E[u]$ |
|---|---|---|---|---|---|---|---|
| 2A | 0.8 | 0.3 | (0.0000, 0.0000) | Mixed | 1.2620/0.5189=2.4321 | 0.9723 | 0.1741 |
| 2B | 0.8 | 0.5 | (0.0000, 0.0000) | Mixed | 1.2947/0.4905=2.6396 | 0.9707 | 0.2328 |
| 2C | 0.8 | 0.7 | (0.0000, 0.0000) | Mixed | 1.3296/0.4646=2.8618 | 0.9700 | 0.2883 |
| 7A | 0.8 | 0.3 | (0.1000, 0.0084) | Mixed | 1.1704/0.5794=2.0200 | 0.9222 | 0.1729 |
| 7B | 0.8 | 0.5 | (0.1000, 0.0069) | Mixed | 1.2190/0.5251=2.3215 | 0.9356 | 0.2318 |
| 7C | 0.8 | 0.7 | (0.1000, 0.0060) | Mixed | 1.2638/0.4854=2.6036 | 0.9431 | 0.2875 |
| 8A | 0.8 | 0.3 | (0.2000, 0.0553) | Mixed | 1.1350/0.5493=2.0663 | 0.7834 | 0.1669 |
| 8B | 0.8 | 0.5 | (0.2000, 0.0339) | Mixed | 1.1836/0.5050=2.3438 | 0.8550 | 0.2281 |
| 8C | 0.8 | 0.7 | (0.2000, 0.0249) | Mixed | 1.2286/0.4701=2.6135 | 0.8894 | 0.2847 |

Table 6: The effects of government deposit insurance: $\phi = \rho\tau/(1-\rho)$.

# 8 Conclusions

This paper have developed a simple banking model in a small open economy in which financial system is opened to include an international capital market. The model explains the basic features of banking crises of emerging economies after financial liberalization: (i) capital inflow increases the probability and the size of a financial crisis; (ii) domestic banks are in a situation of internationally illiquid before a crisis; and (iii) banks' assets are traded at fire-sale prices under a crisis.

Simple examples presented in Section 6 stress the general equilibrium effects of the liquidity regulation and the deposit insurance policy, which most partial equilibrium models are missing. These policies have an impact on banks' portfolio choice, which in turn affects the number of risky banks and the asset prices.

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