

# On a Characterization of the Multiplicative Human Development Index

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## Abstract

The Human Development Index (HDI) assesses the well-being of a society by aggregating people's partial achievements in three fundamental dimensions: health, education, and income. Its aggregation formula was revised in 2010 from the arithmetic mean to the geometric mean. Herrero, Martínez, and Villar (2010) is one of the most influential papers on this revision. They point out that the old HDI deals with partial achievements in the three dimensions as completely substitutable, which is inconsistent with the basic idea behind the HDI: while health and education are ends in themselves, income is just a means to these ends (Anand and Sen 1994). To exclude indices with such a drawback, Herrero, Martínez, and Villar (2010) introduce *minimal lower boundedness*. They claim that a class of multiplicative indices can be axiomatized by *minimal lower boundedness* and other desirable properties. This class includes the new HDI as a special case.

However, we show that their claim does not hold. We find a non-multiplicative index satisfying all the axioms by Herrero, Martínez, and Villar (2010). We also cast doubt on the validity of other results indebted to the axiomatization. In particular, Zambrano (2014)'s Theorem 1 does not hold, which pins down the new HDI with additional axioms.

*Keywords:* Human Development Index, Capabilities, Multiplicative Index

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