## Commodity Spot, Forward Prices, and Convenience Yield under Incomplete Market \*

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## Abstract

This paper analyzes the relation between commodity spot, forward prices, and convenience yield under incomplete market. We model a maximization profit model of a firm that uses input commodities in order to produce output commodities while storing spot commodities and trading forward to hedge its risk. Here we assume the firm have constraints for holding spot commodity. Since the model is stemmed on the incomplete market, we assume that the firm maximizes its profit under a present value vector. Solving the maximization problem, we can relate commodity spot, forward price, and the shadow price which is interpreted as the convenience yield. We enhance the model to include multiple firms and multiple speculators and prove the generic existence of the equilibrium.

Keywords: commodity prices, convenience yield, forward price, incomplete market, production, storage. JEL Classification: C62, D52, G12, G13

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