

Generalized Potentials, Value and Core*

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January 29, 2018

Abstract

Our objective is to analyse the relationship between the Shapley value and the core from the aspect of the potential of a game. To this end, we introduce a new concept, *generalized HM-potential*, which is a generalization of the potential function defined by Hart and Mas-colell (1989). We show that the Shapley value lies in the core if and only if the maximum of the generalized HM-potential of a game is less than a cutoff value. Moreover, we show that this is equivalent to that the minimum of the generalized HM-potential of a game is greater than another different cutoff value. We also provide a geometric characterization of the class of games in which the Shapley value lies in the core, which also shows the relationship with convex games and average convex games as a corollary. Our results suggest a new approach to utilize the potential function in cooperative game theory.

Keywords: Shapley value; Core; Potential; Cooperative game

JEL Classification: C71

* We thank Yukihiro Funaki, Toshiyuki Hirai, Stephen Morris, Takashi Ui, Jun Wako, the seminar participants in the Graduate Summer Workshop on Game Theory 2017 at Hokkaido and Toyama University for helpful comments. Abe and Nakada acknowledge the financial support from the Japan Society for Promotion of Science (JSPS).

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