Quality Dependent Trade Costs and Nonlinearity of Wage Inequality

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Abstract

This paper proposes a theory of wage inequality based on sector specific trade costs. Unskilled labor suffer from higher trade barriers: e.g. due to the low market value of unskilled intensive goods, they must pay relatively higher per unit transport costs. This paper uses this framework to show: (i) the effect of the transportation infrastructure for the time series of wage inequality; (ii) the effect of trade liberalization when low quality goods faces stronger trade barrier than high quality goods. The effect of the reduction in trade barriers is in general nonlinear—e.g. transportation development causes Kuznets curve.

JEL Classification Codes: F16; J31; L91; O11; R1
Keywords: Alchian-Allen Hypothesis; Kuznets Curve; Trade Liberalization; Transportation Infrastructure; Wage Inequality

1 Introduction

Unskilled labors suffer from stronger trade barriers than skilled labors. For example, transport costs depend on the mass of goods. More concretely, consider the economy that the unskilled and the skilled produces one unit of machines. The size and mass of the machine produced by the unskilled equal that of the machine produced by the skilled. The machine of the unskilled, however, has a relatively low quality and a lower market

value (Schott (2004); Khandelwal (2010)).[1] To export the machine of the unskilled, a higher portion of the revenue is seized as transport costs when transport costs depend on the mass of goods (Alchian and Allen (1972); Hummels and Skiba (2004); Irarrazabal et al. (2015); Lugovskyy and Skiba (2015)). This relatively high trade barrier reduces the labor demand and the wage of the unskilled.[2]

The purpose of this paper is to show that sector specific costs have new distributional implications. There are two main questions in this paper. First question is how the development of transportation technologies such as railroad construction affect wage inequality. While previous literature clarified that these infrastructures improve efficiency and is an important source of economic development (Baum-Snow (2007); Donaldson (2010); Donaldson and Hornbeck (2016); Limao and Venables (2001)), this paper shows that transportation technology also affects income distribution. It is true that previous studies also focused on the relationship between transportation and *interregional* inequality (Donaldson (2010); Pascali (2016)). Especially, researchers of economic geography clarified how transportation costs affect interregional asymmetry (Fujita et al. (2001); Ottaviano et al. (2002)). In contrast to previous studies, however, we focus on the *intraregional* inequality. Sector specific transport costs affect income distribution within region.

Second question is that the effect of uniform reduction of trade barriers for wage inequality when there is the sectoral difference of trade barriers. It is true that previous studies suggested that the trade liberalization affects relative demand of skill. Notable example is the literature on skill biased technical change (Acemoglu (2003); Acemoglu et al. (2015); Burstein and Vogel (2017); Epifani and Gancia (2008); Sampson (2014); Yeaple (2005)). In contrast to these studies, however, this paper shows that the sectoral difference of trade barriers produces the relationship between trade liberalization and inequality.

This paper is not new about examining the relationship between trade barriers and relative demand. The implications of trade barriers for *consumption goods demand* is well examined by previous studies (Feenstra (1988); Fajgelbaum et al. (2011)). However, they focused mainly on the *industrial*

[1] Moreover, many theoretical studies often assume this positive correlation between product quality and skill intensity (Stokey (1991); Murphy and Shleifer (1997)).

[2] Moreover, the theory of optimal trade policy suggested the sector or firm specific tariff/subsidy (Costinot et al. (2015, 2016)).

structure. What is new to this paper is to focus on the *relative factor demand*. The change in trade costs has many intriguing distributional effects.

Main results of this paper can be summarized as follows: (i) Transportation development causes *Kuznets curve* (Kuznets (1955); Barro (2000)). That is, as transport technology improves, since only skilled intensive goods can trade, wage inequality increases at first and then, as unskilled intensive goods also trade, wage inequality decreases. Interestingly, this result can explain the observation of Banerjee et al. (2012): access to transportation network increases the income of median household whereas that of the poorest and the richest household decreases. A policy implication of this result is that the intensive investment for transportation infrastructure not only enhances efficiency but also can alleviate wage inequality. (ii) The change of income inequality after trade liberalization crucially depend on the relative *import shares* (Arkolakis et al. (2012, 2015)). Therefore, wage inequality increases after trade liberalization when unskilled intensive goods suffer from high trade barriers. This itself is a expected result. Nevertheless, There is an additional implication: *non-linerality* of the effect of trade liberalization. As trade liberalizes, the effect of liberalization magnifies. More concretely, if the import share or the trade volume is low, the effect of trade liberalization is low. As the trade volume expands, the effect of the trade becomes non-negligible. This result implies that the evaluation of the effect of trade liberalization by previous liberalization is problematic. Even when previous liberalization does not affect wage inequality, this does not imply that additional liberalization does not change wage inequality so much.

This paper proceeds as follows. Section 2 states the structure of the model and obtains equilibrium skill premium. Since this paper is based on theoretical interests, this paper does not distinct international trade and interregional trade within a nation. Section 3 examines the effect of trade liberalization—the effect of the uniform reduction of the tariff. Section 4 examines the effect of the transport costs. Section 5 concludes.

2 Model

Consider the economy with $M + 1$ countries. M can be interpreted as the number of trade partners. To concentrate on the sector specific trade costs, we assume that countries are symmetric. One possible justification

for symmetric assumption is that the cross country factor difference is not so important for wage inequality. Indeed, international trades mainly occurs between countries with similar income level (Krugman (2000); Waugh (2010)). Moreover, in contrast to the explanation of income inequality based on interregional factor differences, many empirical studies suggested that even low skill abundant developing countries increased wage inequality after trade liberalization (Goldberg and Pavcnik (2007); Verhoogen (2008)). These studies indicated that the main source of inequality is not the factor difference across countries.

Consumers in each country i have the following utility function U_i:

$$U_i = (Y_{si}^{\frac{\epsilon-1}{\epsilon}} + Y_{ui}^{\frac{\epsilon-1}{\epsilon}})^{\frac{\epsilon}{\epsilon-1}} \tag{1}$$

where Y_{si} and Y_{ui} are consumptions of high quality goods s and that of low quality goods u respectively. $\epsilon > 1$[3] is the elasticity of substitution across the final goods. First order conditions of the utility maximization problem give the following relative expenditure share:

$$\frac{P_{si}Y_{si}}{P_{ui}Y_{ui}} = (\frac{P_{si}}{P_{ui}})^{1-\epsilon} \tag{2}$$

where P_{hi} is the price of the final goods Y_{hi}.

All market are perfect competitive. Nontradable final goods Y_{hi} are produced by tradable intermediate goods. Following Armington (1969), each intermediate goods are assumed to be differentiated across region. The production functions for final goods have the following form:

$$Y_{hi} = (\sum_{j=0}^{M} y_{hij}^{\frac{\rho-1}{\rho}})^{\frac{\rho}{\rho-1}} \tag{3}$$

where y_{hij} is the supply of sector h intermediate goods sold from country j to i. $\rho > 1$[4] is the elasticity of substitution across goods produced by

[3]Assumptions similar to $\epsilon > 1$ were also used for previous literature and justified empirically. For example, Acemoglu (1998) and Acemoglu and Autor (2011) reviewed literature and showed that it is plausible that factor demands for the skilled and the unskilled are gross substitute (i.e. the elasticity of substitution is higher than one). Epifani and Gancia (2008) also suggested that skill and unskilled intensive goods are gross substitute.

[4]This assumption is often taken for avoiding the possibility of immiserizing growth (Acemoglu and Ventura (2002)).

different regions. The prices of final goods is determined by intermediate goods prices:

$$P_{hi} = (\sum_j p_{hij}^{\rho-1})^{\frac{1}{\rho-1}} \tag{4}$$

where p_{hij} is the price of intermediate goods.

Then we consider the production of intermediate goods. According to Epifani and Gancia (2008) and Krugman (1981), each intermediate good is assumed to be produced by sector specific labors: skilled labors s and low skill labors u produce high quality goods and low quality intermediate goods respectively. The prices of intermediate goods is determined as follows. For each sector, unit labor requirement is one. In addition, two types of iceberg trade costs are needed to export other regions. First, $\theta_h \tau_p$ units of iceberg transport cost is used for export. I normalize θ_u to one. The transport cost is low for high quality goods: $\theta_s < \theta_u = 1$. Second, τ_t units of goods is levied as tariff. For simplicity, the tariff is assumed to be wasted and not redistributed for consumers. The gross total trade cost t_h is

$$t_h \equiv (1 + \theta_h \tau_p)(1 + \tau_t) \tag{5}$$

By perfect competition, the price of intermediate goods equals marginal cost:

$$\begin{aligned} p_{hij} &= w_h && i = j \\ &= t_h w_h && i \neq j \end{aligned} \tag{6}$$

Therefore, from (4),

$$P_{hi} = (1 + M t_h^{\rho-1})^{\frac{1}{\rho-1}} w_{hi} \tag{7}$$

where w_h denotes the wage of the sector h of the country i.

Now we have reached the final step to obtain the equilibrium wage premium $\frac{w_s}{w_u}$. To do so, we examine how factor shares are determined. Perfect competition ensures that all revenues are distributed to the input of the goods:

$$\sum_{i=0}^{M} p_{hij} y_{hij} = w_h L_h$$

$$P_{hi} Y_{hi} = \sum_{j=0}^{M} p_{hij} y_{hij}$$

and therefore

$$P_{hi}Y_{hi} = w_h L_h \tag{8}$$

where L_h is the within-region endowment of the labor h. For deriving the second equality, we also used the assumption that all trade costs are wasted. From (2) and (8), the relative factor is inversely correlated with the relative final goods prices:

$$\frac{w_{si}L_s}{w_{ui}L_u} = (\frac{P_{si}}{P_{ui}})^{1-\epsilon} \tag{9}$$

Now we can obtain the equilibrium skill premium $\omega = \frac{w_s}{w_u}$. From (2) and (7),

$$\omega^{\epsilon} = (\frac{L_u}{L_s})(\frac{1+\hat{M}(1+\theta\tau_p)^{1-\rho}}{1+\hat{M}(1+\tau_p)^{1-\rho}})^{\frac{\epsilon-1}{\rho-1}} \tag{10}$$

where $\hat{M} \equiv M(1+\tau_t)^{1-\rho}$ is a measure of openness, which is large when tariff is low or there are many trade partners.

How do transport costs affect skill premium? We start from viewing the asymptotic behavior of ω:

$$\inf_{\tau_p,\hat{M}} \omega = \lim_{\tau_p \to 0} \omega = \lim_{\tau_p \to \infty} \omega \tag{11}$$

$$= (\frac{L_u}{L_s})^{1/\epsilon} \tag{12}$$

The relationship here has two implications. First, $\lim_{\tau_p \to 1} \omega = \lim_{\tau_p \to \infty} \omega = \lim_{\theta \to 1} \omega$ indicates that cost difference matters only when the transport cost has moderate size. This is because transport cost differences are not matter when trade is prohibited or transport costs are negligible. Second, $\omega(\theta_s, \tau_p, \tau_t) > \lim_{\tau_p \to \infty} \omega = \lim_{\theta_s \to 1} \omega$ indicates that the transport cost has a nonlinear effect on skill premium. This nonlinearity is formally examined in Section 4.

3 Trade Liberalization

In this section, we show the effect of trade liberalization $d\hat{M} > 0$. Trade liberalization is caused by the rise of the number of trade partners $dM > 0$

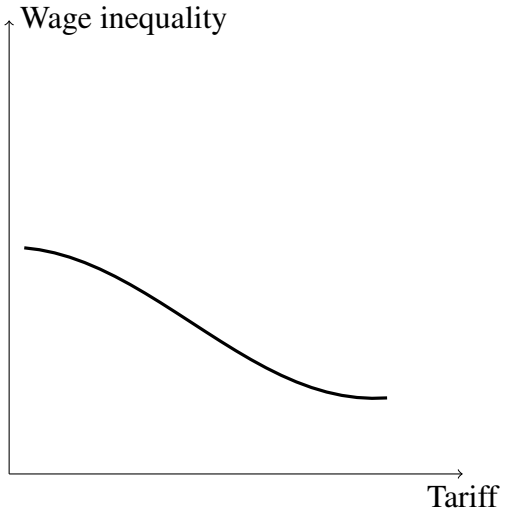

Figure 1: Trade liberalization and wage inequality.

or the uniform reduction of tariff $d\tau_t < 0$. Trade liberalization worsens wage inequality (See also Figure 1):

$$\frac{d\ln\omega}{d\ln\hat{M}} = \frac{\epsilon-1}{\epsilon(\rho-1)}\left(\frac{\hat{M}(1+\theta_s\tau_p)^{1-\rho}}{1+\hat{M}(1+\theta_s\tau_p)^{1-\rho}} - \frac{\hat{M}(1+\tau_p)^{1-\rho}}{1+\hat{M}(1+\tau_p)^{1-\rho}}\right) > 0 \qquad (13)$$

where $\dfrac{\hat{M}(1+\theta_h\tau_p)^{1-\rho}}{1+\hat{M}(1+\theta_h\tau_p)^{1-\rho}}$ is the sectoral import share. The trade liberalization is more beneficial for the labor who works in the sector more goods is exported. Gains from trade can be measured by the import share (Arkolakis et al. (2012, 2015)). Moreover, the positive correlation between the export share and the skill intensity is roughly consistent with the empirical studies. Many papers suggested that the skill intensity of export firms is higher than that of non export firms (Bernard and Jensen (1995); Bernard et al. (2012)).

Moreover, the positive relationship between the sectoral import share and the sectoral gains from trade has much more implications. To see this, suppose that trade costs is prohibitively high. In this case, trade liberalization has only a negligible impact: $\lim_{\hat{M}\to 0}\frac{d\ln\omega}{d\ln\hat{M}} = 0$. Roughly speaking, if the trade volume is small, the effect of trade for wage inequality is also negligible. As the trade volume expands by trade liberalization, the effect

of the interregional trade becomes non-negligible. Indeed, the sectoral import share of high quality goods is lower than 50 percent, we can verify that the size of the effect of trade liberalization expands: $\frac{d^2 \ln \omega}{d(\ln \hat{M})^2} > 0$.[5] This implies that even if previous liberalization does not worsen wage inequality, additional liberalization can worsen wage inequality. Hence, researchers must carefully handles with information from previous liberalization to evaluate the effect of liberalization.

The main feature of this model is that it is quite different from neoclassical factor endowment theory. Factor endowment theory said that the relative position of unskilled labors worsens trade with unskilled abundant countries. To alleviate wage inequality, they recommended trade barriers for the unskilled intensive sector. Here, on the contrary, higher trade barriers of low skilled intensive goods produce the relative low wage of unskilled labors. The relative higher trade barrier of the unskilled intensive sector hampers division of labor of unskilled labors. Hence, it also worsens the relative position of unskilled labors.

Now we discuss an another interpretation of the increment of the trade partners $dM > 0$. This can be interpreted as the rise of the emerging countries such as China. As we have stated, the international trade occurs between similar countries. As time passes, however, emerging countries becomes similar to developed countries. Then, the trade volume between developed countries and emerging countries has increased and the rise of emerging countries cause wage inequality of developed countries. This story roughly matches the recent event (Autor et al. (2013)). Admittedly, this is an extremely crude interpretation and must be viewed at most as a first order approximation. This interpretation, however, delivers a new insight for the recent event.

4 Transportation Costs and Kuznets Curve

In this section, we examine the effect of the transportation technology $d\tau_p < 0$. The development of the transport technology has a nonlinear

[5] See appendix.

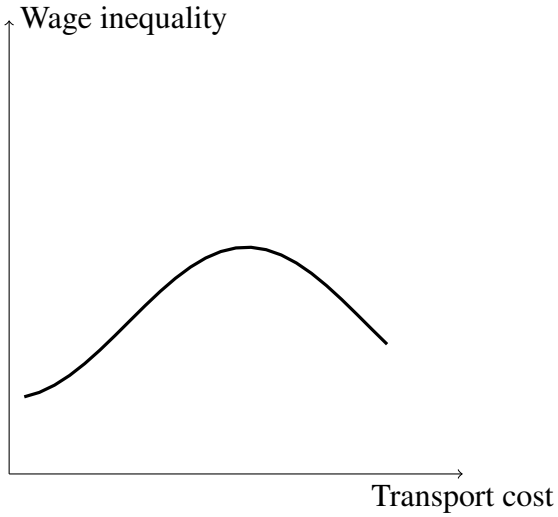

Figure 2: Transport cost and wage inequality.

effect on wage inequality:

$$\begin{aligned}\frac{d\ln\omega}{d\ln\tau_p} = \frac{\epsilon-1}{\epsilon}\frac{1+\hat{M}(1+\tau_p)^{1-\rho}}{1+\hat{M}(1+\theta\tau_p)^{1-\rho}}\frac{\hat{M}\tau_p(1+\tau_p)^{\rho}}{(1+\theta\tau_p)^{\rho}} \\ \times \frac{(1+\theta\tau_p)^{\rho}-\theta(1+\tau_p)^{\rho}+(1-\theta)\hat{M}}{((1+\tau_p)^{\rho}+(1+\tau_p)\hat{M})^2}\end{aligned} \tag{14}$$

The behavior of $\omega(\tau_p)$ is similar to Kuznets curve (See Figure 2). Inverted U shaped behavior of $\omega(\tau_p)$ is confirmed because $(1+\theta\tau_p)^{\rho}-\theta(1+\tau_p)^{\rho}+(1-\theta)\hat{M}$ is a decreasing function to τ_p. Intuitively, when transport costs are sufficiently high, the reduction in transport costs is beneficial only for skilled labors, who suffer from relatively low trade barriers. Then as transport cost is decreasing, unskilled labors also gains from trade.

A policy implication of this result is that sufficiently large investments for transportation infrastructures can alleviate wage inequality.

As I know, it is a new theoretical mechanism of Kuznets curve. The inverted U shaped curve is obtained by the disproportional gains from transportation infrastructures. It was well argued that the reduction of trade barriers facilitates division of labor (Smith (1827)). Here, we show that the gains from trade is nonuniform to labors. It produces many distributional conflicts.

5 Conclusion

This paper begins with the empirical observations that the transport cost of the high quality goods is relatively low and that the high quality goods are produced by the skilled labors. Although the model in this paper is quite simple, many insights can be obtained: (i) The development of the transportation infrastructure produces Kuznets curve. (ii) Trade liberalization increases wage inequality and it's effect is also increasing.

Moreover, these results have many policy implications. First, since the effect of the trade liberalization is nonlinear, the information from the previous liberalization may not be informative. Second, investments in the transportation infrastructure can change wage inequality. Specially, if the size of the investment is sufficiently large, it can reduces wage inequality.

The analysis in this paper depends on various assumptions. Especially, it relies on the iceberg specification of trade costs.[6] Admittedly, the analysis so far is not conclusive. Despite the limitation, we make progress and show that this new framework delivers many new viewpoints.

References

Acemoglu, Daron, "Why do new technologies complement skills? Directed technical change and wage inequality," *Quarterly journal of economics*, 1998, *113* (4), 1055–1089.

_ , "Patterns of skill premia," *Review of Economic Studies*, 2003, *70* (2), 199–230.

_ **and David Autor**, "Skills, tasks and technologies: Implications for employment and earnings," *Handbook of labor economics*, 2011, *4*, 1043–1171.

_ **and Jaume Ventura**, "The world income distribution," *Quarterly Journal of Economics*, 2002, *117* (2), 659–694.

[6]One difficulty specific to this paper is that in contrast to previous literature, there are two factors of production in the model here. Therefore, what factor of production is used for transport costs is not a trivial problem. One possible extension to make the model more realistic is to incorporate the explicit transportation sector in the model (Burstein et al. (2003); Matsuyama (2007)).

_ , **Gino Gancia, and Fabrizio Zilibotti**, "Offshoring and directed technical change," *American Economic Journal: Macroeconomics*, 2015, *7* (3), 84–122.

Alchian, Armen Albert and William Richard Allen, "University economics", Wadsworth Publishing Company; 3rd edition, 1972.

Arkolakis, Costas, Arnaud Costinot, and Andrés Rodríguez-Clare, "New trade models, same old gains?," *American Economic Review*, 2012, *102* (1), 94–130.

_ , _ , **Dave Donaldson, and Andrés Rodríguez-Clare**, "The elusive pro-competitive effects of trade," *NBER Working Paper*, 2015.

Armington, Paul S, "A theory of demand for products distinguished by place of production," *IMF Staff Papers*, 1969, *16* (1), 159–178.

Autor, David H, David Dorn, and Gordon H Hanson, "The China syndrome: Local labor market effects of import competition in the United States," *American Economic Review*, 2013, *103* (6), 2121–2168.

Banerjee, Abhijit, Esther Duflo, and Nancy Qian, "On the road: Access to transportation infrastructure and economic growth in China," *NBER Working Paper*, 2012.

Barro, Robert J, "Inequality and Growth in a Panel of Countries," *Journal of economic growth*, 2000, *5* (1), 5–32.

Baum-Snow, Nathaniel, "Did highways cause suburbanization?," *The Quarterly Journal of Economics*, 2007, *122* (2), 775–805.

Bernard, Andrew B and J Bradford Jensen, "Exporters, jobs, and wages in US manufacturing: 1976-1987," *Brookings papers on economic activity*, 1995, *1995*, 67–119.

_ , _ , **Stephen J Redding, and Peter K Schott**, "The empirics of firm heterogeneity and international trade," *Annual Review of Economics*, 2012, *4* (1), 283–313.

Burstein, Ariel and Jonathan Vogel, "International trade, technology, and the skill premium," *Journal of Political Economy*, 2017, *125* (5), 1356–1412.

Burstein, Ariel T, Joao C Neves, and Sergio Rebelo, "Distribution costs and real exchange rate dynamics during exchange-rate-based stabilizations," *Journal of monetary Economics*, 2003, *50* (6), 1189–1214.

Costinot, Arnaud, Andrés Rodríguez-Clare, and Iván Werning, "Micro to macro: Optimal trade policy with firm heterogeneity," *NBER Working Paper*, 2016.

_ , Dave Donaldson, Jonathan Vogel, and Iván Werning, "Comparative advantage and optimal trade policy," *The Quarterly Journal of Economics*, 2015, *130* (2), 659–702.

Donaldson, Dave, "Railroads of the Raj: Estimating the impact of transportation infrastructure," *NBER Working Paper*, 2010.

_ and Richard Hornbeck, "Railroads and American economic growth: A " market access " approach," *Quarterly Journal of Economics*, 2016, *131* (2), 799–858.

Epifani, Paolo and Gino Gancia, "The Skill Bias of World Trade," *Economic Journal*, 2008, *118* (530), 927–960.

Fajgelbaum, Pablo, Gene M Grossman, and Elhanan Helpman, "Income distribution, product quality, and international trade," *Journal of political Economy*, 2011, *119* (4), 721–765.

Feenstra, Robert C, "Quality change under trade restraints in Japanese autos," *The Quarterly Journal of Economics*, 1988, *103* (1), 131–146.

Fujita, Masahisa, Paul Krugman, and Anthony J Venables, *The spatial economy: Cities, regions, and international trade*, MIT press, 2001.

Goldberg, Pinelopi Koujianou and Nina Pavcnik, "Distributional effects of globalization in developing countries," *Journal of economic Literature*, 2007, *45* (1), 39–82.

Hummels, David and Alexandre Skiba, "Shipping the good apples out? An empirical confirmation of the Alchian-Allen conjecture," *Journal of political Economy*, 2004, *112* (6), 1384–1402.

Irarrazabal, Alfonso, Andreas Moxnes, and Luca David Opromolla, "The tip of the iceberg: a quantitative framework for estimating trade costs," *Review of Economics and Statistics*, 2015, *97* (4), 777–792.

Khandelwal, Amit, "The long and short (of) quality ladders," *The Review of Economic Studies*, 2010, *77* (4), 1450–1476.

Krugman, Paul, "Technology, trade and factor prices," *Journal of international Economics*, 2000, *50* (1), 51–71.

Krugman, Paul R, "Intraindustry specialization and the gains from trade," *Journal of political Economy*, 1981, *89* (5), 959–973.

Kuznets, Simon., "Economic growth and income inequality," *American economic review*, 1955, *45* (1), 1–28.

Limao, Nuno and Anthony J Venables, "Infrastructure, geographical disadvantage, transport costs, and trade," *World Bank Economic Review*, 2001, *15* (3), 451–479.

Lugovskyy, Volodymyr and Alexandre Skiba, "How geography affects quality," *Journal of Development Economics*, 2015, *115*, 156–180.

Matsuyama, Kiminori, "Beyond icebergs: towards a theory of biased globalization," *Review of Economic Studies*, 2007, *74* (1), 237–253.

Murphy, Kevin M and Andrei Shleifer, "Quality and trade," *Journal of development economics*, 1997, *53* (1), 1–15.

Ottaviano, Gianmarco, Takatoshi Tabuchi, and Jacques-François Thisse, "Agglomeration and trade revisited," *International Economic Review*, 2002, pp. 409–435.

Pascali, Luigi, "The wind of change: Maritime technology, trade and economic development," *American Economic Review*, 2016.

Sampson, Thomas, "Selection into trade and wage inequality," *American Economic Journal: Microeconomics*, 2014, *6* (3), 157–202.

Schott, Peter K, "Across-product versus within-product specialization in international trade," *Quarterly Journal of Economics*, 2004, *119* (2), 647–678.

Smith, Adam, *An Inquiry into the Nature and Causes of the Wealth of Nations* number 25202, Printed at the University Press for T. Nelson and P. Brown, 1827.

Stokey, Nancy L, "Human capital, product quality, and growth," *Quarterly Journal of Economics*, 1991, *106* (2), 587–616.

Verhoogen, Eric A, "Trade, quality upgrading, and wage inequality in the Mexican manufacturing sector," *Quarterly Journal of Economics*, 2008, *123* (2), 489–530.

Waugh, Michael, "International trade and income differences," *American Economic Review*, 2010, *100* (5), 2093–2124.

Yeaple, Stephen Ross, "A simple model of firm heterogeneity, international trade, and wages," *Journal of international Economics*, 2005, *65* (1), 1–20.

A The sign of $\frac{d^2 \ln \omega}{d(\ln \hat{M})^2}$

We start from the second derivative of $\omega(\hat{M})$:

$$\frac{d^2 \ln \omega}{d(\ln \hat{M})^2} = \frac{\epsilon - 1}{\epsilon(\rho - 1)} \{ \frac{\hat{M}(1+\theta_s \tau_p)^{1-\rho}}{(1+\hat{M}(1+\theta_s \tau_p)^{1-\rho})^2} - \frac{\hat{M}(1+\tau_p)^{1-\rho}}{(1+\hat{M}(1+\tau_p)^{1-\rho})^2} \} \quad (15)$$

Therefore, $\frac{d^2 \ln \omega}{d(\ln \hat{M})^2}$ is positive if and only if

$$\frac{\hat{M}(1+\theta_s \tau_p)^{1-\rho}}{(1+\hat{M}(1+\theta_s \tau_p)^{1-\rho})^2} > \frac{\hat{M}(1+\tau_p)^{1-\rho}}{(1+\hat{M}(1+\tau_p)^{1-\rho})^2} \quad (16)$$

What we need to verify is that if the skilled import share $\frac{\hat{M}(1+\theta_s \tau_p)^{1-\rho}}{(1+\hat{M}(1+\theta_s \tau_p)^{1-\rho})}$ is lower than $1/2$, then (16) is satisfied. Now we are prepared for verifying this statement. To do so, we introduce the following functions:

$$\lambda(x) = \frac{x}{1+x} \quad (17)$$

$$\phi(x) = \frac{x}{(1+x)^2} \quad (18)$$

We start from the meaning of λ. It intimately linked to the import shares. Indeed, $\lambda(\hat{M}(1+\theta_s\tau_p)^{1-\rho})$ is the skilled import share. The following properties of λ are easily derived:

- $\lambda(x)$ is monotonically increasing to x.
- $\lambda(x) < 1/2$ if and only if $x < 1$

Moreover, from the second fact, the skilled import share $\lambda(\hat{M}(1+\theta_s\tau_p)^{1-\rho})$ is lower than $1/2$ if and only if $\hat{M}(1+\theta_s\tau_p)^{1-\rho}$ is lower than one.

Then we show the meaning of ϕ. (16) is equivalent to

$$\phi(\hat{M}(1+\theta_s\tau_p)^{1-\rho}) > \phi(\hat{M}(1+\tau_p)^{1-\rho}) \tag{19}$$

Now we only need to show that $\phi(x)$ is monotonically increasing in the region $x \in (0,1)$. Indeed, since $\hat{M}(1+\tau_p)^{1-\rho} < \hat{M}(1+\theta_s\tau_p)^{1-\rho} < 1$, then $\frac{d\phi}{dx} > 0$ ensures that (19) is satisfied. Since $\frac{d\phi(x)}{dx} = \frac{1-x}{(1+x)^2}$, we obtain

- $\frac{d\phi(x)}{dx} > 0$ if $x < 1$

Hence, we have shown that (16) is satisfied.