Joint Distribution of City and Average Establishment Sizes

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Abstract

This paper examines the mechanism behind the fact that the size of a city and the average establishment size in the city are positively correlated. It also examines the dependence of the size distribution of cities on industrial properties, which are crucial in determining the size distribution of firms (establishments). For these purposes, we introduce monopolistic competition to the stochastic urban growth model by Rossi-Hansberg and Wright (2007b, "Urban Structure and Growth," *Review of Economic Studies* 74, pp.597-624.) in order to endogenously determine the size distribution of establishments as well as that of cities. The model, calibrated to match the U.S. economy, can reproduce both the size distribution of cities and the joint distribution of city and average establishment sizes except for generating variation of the establishment size more volatile than data. Counterfactual simulations suggest that, in an empirically relevant situation, the size distribution of cities is almost independent of the distribution of operating costs and productivities of industries, while the contrasting result holds in the case of the joint distribution of city and average establishment sizes. Self selection and a lower skewed distribution of operating costs and productivities of industries are key elements in generating the observed joint distribution.

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1. Introduction

There are two strands in the literature on the size distribution of economic units: cities and firms (establishments). The upper tail of the size distribution of cities is well approximated by a Pareto distribution.[1] The size distribution of all firms is also well approximated by a Pareto distribution with coefficient one. Although researchers have already reached a consensus on the statistical mechanism behind these phenomena, economic interpretation is still an open question (Gabaix and Ioannides, 2004; Gabaix, 2009; Sutton, 1997; Gabaix, 2009; Luttmer, 2010).

The typical, common approach of theorists of these studies is that the size distribution of cities (firms/establishments) is studied independently of the size distribution of firms (cities) in the urban (firm/establishment dynamics) literature. On the other hand, the main objective of this paper is to consider the sizes of both cities and establishments simultaneously in order to investigate how and to what extent these are related. This is in line with the suggestion by Gabaix and Ioannides (2004).

In addition to examining the independence, our research is also motivated by the fact that the size of a city, measured by population, and the average size of establishments in it, measured by the number of employees per establishment, are positively correlated (Figure 1). Since the average size of establishments differs across industries according to the Statistics of U.S. Businesses (SUSB); and since the industrial diversity in a city, defined by the number of industries the city hosts, is positively correlated with its population (Mori et al., 2008), it is suggested that larger (smaller) cities tend to host industries, the average establishment size of which is relatively large (small). Then, it is interesting to ask what is a quantitative model which can explain this phenomenon.

Following Rossi-Hansberg and Wright (2007b), we build a dynamic stochastic general equilibrium growth model with multiple industries, in which the joint distribution of city and average establishment sizes is determined endogenously. Intermediate goods sectors are characterized by monopolistic competition a la Dixit and Stiglitz (1977). Establishment-level increasing returns to scale in production is the key factor for the determination of the size distribution of establishments. The establishment-level increasing returns then leads to the scale economy at the city level. The size distribution of cities is determined by the trade-off between this and the commuting cost. In the determination of the city size, competitive developers, introduced by Henderson (1974), play a crucial role.

The analytical results we obtain are the same as those of Rossi-Hansberg and Wright (2007b) except for obtaining the total factor productivity (TFP) of a city as a function of micro variables and parameters. The market equilibrium and the efficient allocation are equivalent because of the existence of competitive developers. The economy as a whole exhibits constant returns to scale at the macro level. The size distribution of cities deviates from a Pareto distribution in an empirically observed way; that is the size distribution exhibits underrepresentation of both small and very large cities than the Pareto distribution as mentioned by Gabaix and Ioannides (2004).

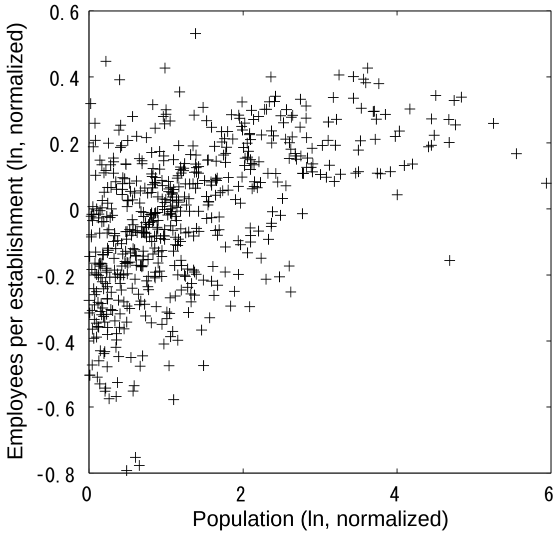

Figure 1: City and Establishment Sizes in the U.S., 2008

The main contribution of this paper is the results of our quantitative analysis. The model, calibrated to match the U.S. economy, can reproduce both the size distribution of the Metropolitan Statistical Areas (MSAs) in the U.S. and the *joint distribution* of city and average establishment sizes, the relationship depicted in Figure 1, under plausible values of parameters except for generating variation of the establishment size more volatile than data.

Counterfactual simulations show that the size distribution of cities depends on industrial properties such as the distribution of operating costs and productivities of industries in general. However, there is a range of distributions, where the size distribution of cities is almost independent of industrial properties. Especially, an empirically plausible distribution of operating costs and productivities is included this range.

Contrary to the case of the size distribution of cities, the joint distribution of city and average establishment sizes depends crucially on industrial properties. Different distributions of operating costs and productivities of industries, which generate almost exactly the same size distribution of cities, can lead to contrasting results on the relationship between the city size and the average establishment size in the city. An empirically plausible distribution is characterized by the one, which is skewed to the lower level and has self selection within industries. Large industries, characterized by higher operating costs and thus higher average establishment sizes, have higher productivities thanks to self selection. TFPs of cities are determined in a way that such positive correlation between the operating cost and the productivity is not eliminated and results in the positive correlation between the operating cost and the TFP. Then, cities, which host larger industries, are able to expand

[1] The coefficient of the Pareto distribution depends on the context, especially the country we focus (e.g. Soo, 2005). For the United States, it is well known that the coefficient is close to one.

even though congestion costs increase.

This paper is related to three literatures. The first is the size distribution of cities. Studies such as Gabaix (1999) and Cordoba (2008) seek economic explanations as to why the upper tail of the distribution is well approximated by a Pareto distribution. Eeckhout (2004) extends the analysis to the case of all settlements.[2] Recently, Duranton (2007) and Rossi-Hansberg and Wright (2007b) provide endogenous growth models, in which the city size distribution exhibits underrepresentation of both small and very large cities than a Pareto distribution. The study of Hsu (2010), who provides a deterministic explanation based on the central place theory, is one of a few studies which consider the size distributions of both cities and firms simultaneously.

The second is the size distribution of firms or establishments. The literature is vast and surveyed by Sutton (1997), Gabaix (2009), and Luttmer (2010).[3] One line of research is based on a class of models which incorporate self selection a la Hopenhayn (1992) and monopolistic competition a la Dixit and Stiglitz (1977) (e.g. Luttmer, 2007). Our model also belongs to this class. However, we simplify the model by assuming that there is no fixed cost and that self selection within industries is imposed exogenously. Like Rossi-Hansberg and Wright (2007a), we focus on the size distribution of establishments not firms.

The third literature is urban growth. Studies such as Eaton and Eckstein (1997), Black and Henderson (1999), and Rossi-Hansberg and Wright (2007b) are included in this literature. The first two provide city growth models under deterministic environments, while the third considers a city growth model with aggregate technology shocks. Our model, though similar to that of the third one, abstracts human capital, the common feature of both lines of research, from the model in order to simplify the discussion.

The structure of this paper is as follows: In Section 2, the environment of the economy, problems of agents, and definition of a market equilibrium are described. Then, Section 3 discusses the analytical properties of a market equilibrium. The results of the quantitative analyses such as calibration and counterfactual simulations are provided in Section 4. Section 5 concludes this paper.

2. The Model

2.1. Environment

We consider a closed economy, which consists of a finite number of industries and an uncountably infinite number of cities. The homogeneous final good and the differentiated intermediate goods sector are vertically linked within each industry. Economic agents consist of households and developers in addition to firms in these two sectors.[4] Production takes place in the central business districts (CBDs) of cities. Time t is discrete; that is $t \in \{0, 1, 2, \cdots\}$.

2.2. Cities

A city is a disk with a CBD at its center. The CBD is expressed as a point in the two dimensional space, and all the production activities in a city takes place within the CBD. Thus, the residential area is equal to the area of the disk. Assuming that each individual consumes one unit of land for housing services, the boundary $\bar{d}$ of a city with population $\tilde{N}$ is determined by the resource constraint: $\pi\bar{d}^2 = \tilde{N}$.

Residents commute to the CBD with the commuting cost which depends on their distance from the CBD. For a resident residing at the distance d from the CBD, the associated commuting cost is τd^γ, where $\gamma > 0$ and $\tau > 0$.[5] Thus, the total commuting cost in a city with population $\tilde{N}$ is

$$TCC = \int_0^{\bar{d}} 2\pi d(\tau d^\gamma)\mathrm{d}d = b\tilde{N}^{\tilde{\gamma}},$$

where $b = (2\tau)/(\gamma + 2)\pi^{-\frac{\gamma}{2}}$ and $\tilde{\gamma} = \gamma/2 + 1$. Dividing TCC by population $\tilde{N}$ gives the average commuting cost:

$$ACC = b\tilde{N}^{\tilde{\gamma}-1}. \quad (1)$$

The urban costs include the land rent in addition to the commuting cost. Let $R(d)$ denote the land rent at the distance d from the CBD. Then, for any given level of labor income, the equilibrium total income net of the commuting cost and the land rent for any two different locations must be equalized; that is $R(d_1) + \tau d_1^\gamma = R(d_2) + \tau d_2^\gamma$ for all $d_1, d_2 \in [0, \bar{d}]$. Normalizing the productivity outside the city to zero, we obtain $R(d) = \tau(\bar{d}^\gamma - d^\gamma)$. Using this, the total land rent TR and average land rent AR are computed as follows:

$$TR = \int_0^{\bar{d}} 2\pi dR(d)\mathrm{d}d = b(\tilde{\gamma} - 1)\tilde{N}^{\tilde{\gamma}}, \quad (2)$$

$$AR = \frac{TR}{\tilde{N}} = b(\tilde{\gamma} - 1)\tilde{N}^{\tilde{\gamma}-1}. \quad (3)$$

We assume that the commuting cost and land rent are measured in terms of the final good produced in the city.

2.3. Final Good Firms

There is a continuum of identical final good firms on the $[0, 1]$ interval. The firms produce a homogeneous good by using only differentiated intermediate goods produced within the city they locate. Production technologies are the same across firms and exhibit constant returns to scale.

[2] Although Eeckhout (2004) argues that the size distribution is well approximated by a lognormal distribution, there is another argument which discusses that the size distribution of all settlements is well approximated by a "double Pareto-lognormal" distribution (e.g. Reed, 2002; Giesen et al., 2010). The double Pareto-lognormal distribution is a distribution, the density of which is lognormal in the middle and Pareto in both the lower and upper tails.

[3] Luttmer (2010) surveys general equilibrium models.

[4] In this paper, we use establishments and firms interchangeably.

[5] This form of the commuting cost is used by Venables (2007).

Formally, a typical final good firm residing in a city which hosts industry $j \in \{1, 2, \cdots, J\}$ solves[6]

$$\max_{\{x_{jt}(v)\}_{v\in[0,\tilde{M}_{jt}]}} \left\{ P_{jt} \left[\int_0^{\tilde{M}_{jt}} (x_{jt}(v))^{\frac{\sigma-1}{\sigma}} \mathrm{d}v \right]^{\frac{\sigma}{\sigma-1}} - \int_0^{\tilde{M}_{jt}} p_{jt}(v) x_{jt}(v) \mathrm{d}v \right\}, \quad (4)$$

where P_{jt} denotes the industry-j final good's price, $p_{jt}(v)$ the variety-v intermediate good's price, $\tilde{M}_{jt}$ the number of varieties within the city, $x_{jt}(v)$ the demand for the variety-v intermediate good of the firm, and $\sigma > 1$ the elasticity of substitution between intermediate goods. For later use, let $\tilde{Y}_{jt}$ denote the city's output of the final good.

Letting c_{jt} denote the cost of production, that is $c_{jt} = \int_0^{\tilde{M}_{jt}} p_{jt}(v) x_{jt}(v) \mathrm{d}v$, the demand for the variety-v intermediate good is given by

$$x_{jt}(v) = \frac{p_{jt}(v)^{-\sigma}}{P_{jt}^{1-\sigma}} c_{jt}, \quad (5)$$

where the price index P_{jt} is given by

$$P_{jt} = \left[\int_0^{\tilde{M}_{jt}} p_{jt}(v)^{1-\sigma} \mathrm{d}v \right]^{\frac{1}{1-\sigma}}. \quad (6)$$

2.4. Intermediate Good Firms

Intermediate good firms are monopolistically competitive. Each firm produces a differentiated good by using physical capital $k_{jt}(v)$ and labor $n_{jt}(v)$ rented from households. The production technology exhibits increasing returns to scale:[7]

$$A_{jt}(k_{jt}(v))^{\theta_j}(n_{jt}(v))^{1-\theta_j} - \phi_j(k_{jt}(v))^{\xi_j} \geq x_{jt}(v), \quad (7)$$

where A_{jt} is the productivity specific to industry j, $\theta_j \in (0, 1)$ the parameter representing importance of capital in industry j, ϕ_j the shift parameter of operating cost for industry j, and $\xi_j \in (0, 1)$ the curvature parameter of the operating cost. We assume that the productivity grows at some rate g_A on an average and is subject to the industry-specific productivity shock z_{jt}:

$$A_{jt} = e^{z_{jt}}(1 + g_A)^t A_{j0}, \quad (8)$$

where $\{z_{jt}\}_t$ follows an AR(1) process:

$$z_{jt} = \rho_j z_{j,t-1} + \varepsilon_{jt}.$$

We assume that the productivity shock is persistent to some extent, that is $\rho_j \in (0, 1)$, and that the innovation ε_{jt} is independent across industries and over time and follows a normal distribution $N(0, \sigma^2_{\varepsilon_j})$.

[6] In the model, each city hosts only one industry. This is because there is no interaction across industries. Since this also applies to the intermediate goods sector, increasing returns to scale in that sector results in specialization of cities.

[7] A similar production technology is assumed by Hornstein (1993). However, the operating cost does not depend on capital stock; that is $\xi_j = 0$.

We also assume that there are no entry and exit costs in the intermediate goods sector. Together with the rental market of capital services, this means that it is sufficient for firms to maximize the current profit in order to maximize the expected discounted profit since a "timing problem" does not exist.

The intermediate goods sector differs from the final goods sector in that the developers discussed later give two types of subsidies to the intermediate good firms. The first is a subsidy for renting capital services, where the rental price r_{jt} of capital is discounted by the subsidy rate τ^k_{jt}. The second is a subsidy for operations. Each developer gives a transfer $T^M_{jt}(v)$, measured in terms of the industry-j final good, to the variety-v intermediate good firm.

Formally, the variety-v intermediate good firm solves

$$\max_{p_{jt}(v), k_{jt}(v), n_{jt}(v)} \pi_{jt}(v) = p_{jt}(v)x_{jt}(v) - (1 - \tau^k_{jt}(v))r_{jt}k_{jt}(v) - w_{jt}n_{jt}(v) + P_{jt}T^M_{jt}(v) \quad (9)$$

subject to (7), where w_{jt} is the nominal wage rate specific to industry j. After some calculations, we obtain

$$(1 - \tau^k_{jt}(v))\frac{r_{jt}}{p_{jt}(v)} = \frac{\sigma - 1}{\sigma} \left\{ \theta_j A_{jt} \left[\frac{n_{jt}(v)}{k_{jt}(v)} \right]^{1-\theta_j} - \xi_j \phi_j (k_{jt}(v))^{\xi_j - 1} \right\}, \quad (10)$$

$$\frac{w_{jt}}{p_{jt}(v)} = \frac{\sigma - 1}{\sigma}(1 - \theta_j)A_{jt} \left[\frac{k_{jt}(v)}{n_{jt}(v)} \right]^{\theta_j}, \quad (11)$$

which shows that the marginal productivity conditions are distorted by the existence of monopolistic competition. The maximized real profit $\pi_{jt}(v)/p_{jt}(v)$ is obtained by substituting (7), (10) and (11) into the objective function:

$$\frac{\pi_{jt}(v)}{p_{jt}(v)} = \frac{1}{\sigma} A_{jt}(k_{jt}(v))^{\theta_j}(n_{jt}(v))^{1-\theta_j} - \left(1 - \xi_j \frac{\sigma - 1}{\sigma}\right) \phi_j (k_{jt}(v))^{\xi_j} + \frac{P_{jt}}{p_{jt}(v)} T^M_{jt}(v).$$

Since there is no idiosyncratic shock, we focus on the symmetric case:

$$\pi_{jt}(v) = \pi_{jt}, \quad k_{jt}(v) = k_{jt}, \quad n_{jt}(v) = n_{jt}, \quad p_{jt}(v) = p_{jt}, \quad \tau^k_{jt}(v) = \tau^k_{jt}, \quad \text{and} \quad T^M_{jt}(v) = T^M_{jt}. \quad (12)$$

Free entry leads to the zero-profit condition; that is $\pi_{jt}/p_{jt} = 0$. Consolidating this, (6) and (12) gives us the total lump-sum subsidy $T^M_{jt}\tilde{M}_{jt}$ to intermediate good firms within a city:

$$T^M_{jt}\tilde{M}_{jt} = \tilde{M}_{jt}^{\frac{\sigma}{\sigma-1}} \left[\left(1 - \xi_j \frac{\sigma - 1}{\sigma}\right) \phi_j k_{jt}^{\xi_j} - \frac{1}{\sigma} A_{jt} k_{jt}^{\theta_j} n_{jt}^{1-\theta_j} \right]. \quad (13)$$

2.5. Developers

New cities are created by competitive developers. We assume that a developer has only one city. Developers attract workers, capital and establishments by subsidizing them since all these factors are crucial in determining the developers' profit. Specifically, developers use three types of subsidies, each corresponding to one of labor, capital stock or operations of establishments. The first is the lump-sum transfer to residents. A developer of a city hosting industry j gives the lump-sum transfer T^n_{jt} equally to the city residents. Since a typical worker earns the nominal labor income w_{jt} and must pay the commuting cost AC_{jt} as well as the land rent AR_{jt}, the net real income Ω_{jt} after considering the lump-sum transfer T^n_{jt} is expressed by

$$\Omega_{jt} = \frac{w_{jt}}{P_{jt}} + T^n_{jt} - ACC_{jt} - AR_{jt}, \tag{14}$$

where T_{jt} is measured in terms of the industry-j final good. Developers must take Ω_{jt} as given because of free mobility of workers.

The second type of the subsidy is the one for renting capital services. As previously mentioned, intermediate good firms can rent capital services at the price discounted by τ^k_{jt}. If we let $\tilde{K}_{jt}$ denote the level of capital services used in a city hosting industry j, a developer's real total subsidy for renting capital services is equal to $(\tau^k_{jt} r_{jt}/P_{jt})\tilde{K}_{jt}$.

The third one is the lump-sum transfer to intermediate good firms. Specifically, the developer of a city hosting industry j gives establishments the lump-sum transfer T^M_{jt} equally in order to influence the number of establishments in the city.[8] Thus, if we let $\tilde{M}_{jt}$ denote the number of establishments in the city, the developer's real total subsidies for operations is equal to $T^M_{jt}\tilde{M}_{jt}$, where T^M_{jt} is measured in terms of the industry-j final good.

These three types of subsidies must be financed by the revenue of the developer, which consists of the land rent only. Therefore, taking the prices (P_{jt}, r_{jt}, w_{jt}) and the transfer Ω_{jt} as given, a typical developer solves

$$\max_{T^n_{jt}, \tau^k_{jt}, T^M_{jt}} \left\{ TR_{jt} - T^n_{jt}\tilde{N}_{jt} - \frac{\tau^k_{jt} r_{jt}}{P_{jt}}\tilde{K}_{jt} - T^M_{jt}\tilde{M}_{jt} \right\}$$

subject to (1), (2), (3), (6), (10), (11), (12), (13), (14), and definitions, $k_{jt} = \tilde{K}_{jt}/\tilde{M}_{jt}$ and $n_{jt} = \tilde{N}_{jt}/\tilde{M}_{jt}$.

We assume that there are many potential developers in the economy. Assuming further that there are no costs of entry and exit in this sector, the optimized profit of developers must be zero in an equilibrium.

2.6. Households

There is a continuum of identical households on the $[0, 1]$ interval. The number N_t of workers in each households grows at some rate g_N; that is $N_{t+1} = (1 + g_N)N_t$. Without loss of generality, we normalize the initial population N_0 to one; that is $N_0 = 1$. Each worker's efficiency is identical and normalized to one. Thus, if we let N_{jt} denote the labor supply for industry j, it must hold that

$$\sum_{j=1}^{J} N_{jt} \leq N_t. \tag{15}$$

Households also own industry-specific physical capital stocks and lend them to the intermediate goods sector. Following Rossi-Hansberg and Wright (2007b), we assume that the capital stocks evolve according the following law of motion:[9]

$$K_{j,t+1} = K^{\omega_j}_{jt} I^{1-\omega_j}_{j,t}, \tag{16}$$

where $I_{j,t}$ is the investment specific to industry j, and $\omega_j \in (0, 1)$.

The preference of households is expressed by the expected discounted utility:

$$E_0 \sum_{t=0}^{\infty} \beta^t \left[N_t \sum_{j=1}^{J} \alpha_j \ln\left(\frac{C_{jt}}{N_t}\right)\right], \tag{17}$$

where $\beta \in (0, 1)$ denotes discount factor, C_{jt} household-level industry-j final good, and $\alpha_j \in (0, 1)$ the expenditure parameter for industry j, which satisfies $\sum_{j=1}^{J} \alpha_j = 1$.

A typical household maximizes the expected discounted utility subject to labor resource constraint (15), the law of motion (16) for each type of capital stock, and period-by-period budget constraints:

$$\begin{aligned} &\sum_{j=1}^{J} P_{jt}[C_{jt} + I_{jt} + (ACC_{jt} + AR_{jt})N_{jt}] \\ &\leq \sum_{j=1}^{J} [w_{jt}N_{jt} + r_{jt}K_{jt} + P_{jt}T^n_{jt}N_{jt}], \end{aligned} \tag{18}$$

Since each household is of measure zero, it takes both the average commuting cost ACC_{jt} and average rent AR_{jt} as given.

2.7. Equilibrium

We now define a market equilibrium of the economy. As previously mentioned, we focus on a symmetric equilibrium, where establishments of any given industry behave similarly.

Definition. *A symmetric equilibrium of the economy is the state in which the price system $(P_{jt}, p_{jt}, r_{jt}, w_{jt})$, the quantities $(C_{jt}, N_{jt}, K_{jt}, I_{jt}, x_{jt}, k_{jt})$, the size distribution $(\mu_{jt}, \tilde{N}_{Jt})$ of cities, the size distribution $(\tilde{M}_{jt}, n_{jt})$ of establishments, and the subsidies $(\tau^k_{jt}, T^n_{jt}, T^M_{jt})$ are such that*

1. *given the prices $\{P_{jt}, w_{jt}, r_{jt}\}_j$, the lump-sum transfer $\{T^n_{jt}\}_j$, the average commuting cost $\{ACC_{jt}\}_j$ and the average rent $\{AR_{jt}\}_j$, households maximize the expected discounted utility (17) subject to the budget constraint (18), the labor constraint (15) and the law of motion (16) for capital stock;*
2. *given the prices (P_{jt}, p_{jt}) and the number $\tilde{M}_{jt}$ of varieties of intermediate goods, industry-j final good firms maximize their profit (4);*
3. *given the demand (5), the aggregate price P_{jt}, the technology (7), and the lump-sum transfer T^M_{jt}, industry-j intermediate good firms maximize their profit, (10) and (11);*

[8] Note that the lump-sum transfer reduces the operating cost of establishments.

[9] This equation states that the growth rate of capital stock is determined by the investment ratio $I_{j,t}/K_{j,t}$.

4. *given the net income Ω_{jt} and the firms' optimal behavior, developers maximize their profit;*
5. *the aggregate price P_{jt} is consistent with individual prices (6);*
6. *free entry into the intermediate goods sector (13);*
7. *free entry into the developer's sector;*
8. *the markets clear*

$$\begin{aligned} \textit{(Final good)} \quad & \mu_{jt}\tilde{Y}_{jt} = C_{jt} + I_{jt} + \mu_{jt} b\tilde{N}_{jt}^{\tilde{\gamma}}, \\ \textit{(Intermediate goods)} \quad & A_{jt}k_{jt}^{\theta_j}n_{jt}^{1-\theta_j} - \phi_j k_{jt}^{\xi_j} = x_{jt}, \\ \textit{(Labor)} \quad & N_{jt} = \mu_{jt}\tilde{M}_{jt}n_{jt}, \\ \textit{(Capital)} \quad & K_{jt} = \mu_{jt}\tilde{M}_{jt}k_{jt}. \end{aligned}$$

3. Properties of a Market Equilibrium

In this section, we discuss the analytical properties of a market equilibrium. We first establish that both the market equilibrium and the solution to the social planner's problem are equivalent. Then, noting that the optimal solution is unique, we derive the size dynamics of cities and establishments. Finally, we obtain the analytical properties of the size distribution of cities and establishments.

3.1. Symmetric Efficient Allocation

To prove the equivalence between the market equilibrium and the social planner's problem, we formulate the social planner's problem as follows:

$$\max_{\{C_{jt}, I_{jt}, N_{jt}\}_{t=0}^{\infty}} E_0 \sum_{t=0}^{\infty} \beta^t \left[N_t \sum_{j=1}^{J} \alpha_j \ln\left(\frac{C_{jt}}{N_t}\right)\right]$$

s.t.

$$C_{jt} + I_{jt} = \mu_{jt}(\tilde{Y}_{jt} - b\tilde{N}_{jt}^{\tilde{\gamma}}), \tag{19}$$

$$\tilde{Y}_{jt} = \tilde{M}_{jt}^{\frac{\sigma}{\sigma-1}} \left(A_{jt}k_{jt}^{\theta_j}n_{jt}^{1-\theta_j} - \phi_j k_{jt}^{\xi_j}\right), \tag{20}$$

$$K_{jt} = \mu_{jt}\tilde{K}_{jt}, \tag{21}$$

$$\tilde{K}_{jt} = \tilde{M}_{jt}k_{jt}, \tag{22}$$

$$N_{jt} = \mu_{jt}\tilde{N}_{jt}, \tag{23}$$

$$\tilde{N}_{jt} = \tilde{M}_{jt}n_{jt}, \tag{24}$$

$$\sum_{j=1}^{J} N_{jt} = N_t, \tag{25}$$

$$K_{j,t+1} = K_{jt}^{\omega_j} I_{jt}^{1-\omega_j}.$$

The resource constraint on the industry-j final good is expressed by (19). The right hand side is the number μ_{jt} of cities hosting industry j times the per-city output $(\tilde{Y}_{jt} - b\tilde{N}_{jt}^{\tilde{\gamma}})$ net of commuting cost. (20) is the production function of the final good under the symmetry of intermediate goods sector (12). (21) states that the beginning-of-period capital stock K_{jt} is allocated across cities in the industry, and (22) shows that the city-level capital $\tilde{K}_{jt}$ is allocated across establishments, whose capital inputs are equal to k_{jt}. (23) and (24) are the labor version. (25) is the labor resource constraint.

We call the optimal solution to the social planner's problem the *symmetric efficient allocation*. The following result shows that it is sufficient to focus on the symmetric efficient allocation in order to characterize the long-run properties of the size distribution of cities and establishments in the economy.[10]

Proposition 1. *The symmetric equilibrium allocation and the symmetric efficient allocation are equivalent. In addition, the symmetric efficient allocation is unique.*

The equivalence between the two is due to the existence of competitive developers. Given that each developer can control economic activities of its city through subsidies, the profit maximization of the developers internalizes the externality and corrects the distortion of the economy. Even though developers have such a great influence on the economic activities, free entry eliminates the monopoly rent of them.

Note that the social planner's problem has the following recursive steps:

1. Given the industry-wide capital K_{jt}, labor N_{jt}, and size distribution $(\mu_{jt}, \tilde{N}_{jt})$ of cities, the gross output $\tilde{Y}_{jt}$ in a city is maximized by choosing the number $\tilde{M}_{jt}$ of establishment within the city.
2. Given the industry-wide capital K_{jt}, labor N_{jt}, and the result of the first step, the economy-wide net output $\mu_{jt}(\tilde{Y}_{jt} - b\tilde{N}_{jt}^{\gamma})$ is maximized by choosing the number μ_{jt} of cities hosting a industry.
3. Given the result of the second step, the expected discounted utility is maximized by choosing the allocation of labor $\{N_{jt}\}_{j=1}^{J}$ and investments $\{I_{jt}\}_{j=1}^{J}$.

The first two steps correspond to static problems embedded in the restrictions to the optimization problem, while the last one is dynamic. Each step is discussed in the following subsections.

3.2. Determination of Establishment Size

The optimization problem in the first step is formulated by substituting (22) and (24) into (20):

$$\max_{\tilde{M}_{jt}} \left\{ \tilde{Y}_{jt} = \tilde{M}_{jt}^{\frac{\sigma}{\sigma-1}} \left[\frac{A_{jt}\tilde{K}_{jt}^{\theta_j}\tilde{N}_{jt}^{1-\theta}}{\tilde{M}_{jt}} - \phi_j \left(\frac{\tilde{K}_{jt}}{\tilde{M}_{jt}}\right)^{\xi_j} \right] \right\}.$$

In determining the gross output $\tilde{Y}_{jt}$, the social planner faces a trade-off between "love of variety" $\tilde{M}_{jt}^{\frac{\sigma}{\sigma-1}}$ and increasing returns to scale at the establishment level, as given in the brackets.[11]

After some calculations, we obtain[12]

$$n_{jt} = \left[(\sigma-1)\left(\frac{\sigma}{\sigma-1} - \xi_j\right)\right]^{\frac{1}{1-\xi_j}} \frac{(\tilde{K}_{jt}/\tilde{N}_{jt})^{\frac{\xi_j-\theta_j}{1-\xi_j}}}{(A_{jt}/\phi_j)^{\frac{1}{1-\xi_j}}}. \tag{26}$$

[10]The proof is provided in the appendix.

[11]Given that $\xi_j \in (0,1)$, it is easy to check that $A_{jt}\tilde{K}_{jt}^{\theta_j}\tilde{N}_{jt}^{1-\theta_j}/\tilde{M}_{jt} - \phi_j(\tilde{K}_{jt}/\tilde{M}_{jt})^{\xi_j} > A_{jt}\tilde{K}_{jt}^{\theta_j}\tilde{N}_{jt}^{1-\theta_j}/(a\tilde{M}_{jt}) - \phi_j[\tilde{K}_{jt}/(a\tilde{M}_{jt})]^{\xi_j}$ for any constant $a \in (0,1)$.

[12]The optimal number $\tilde{M}_{jt}$ of establishments is given by $\tilde{M}_{jt} = \tilde{N}_{jt}/n_{jt}$.

This implies the following: At the establishment level, labor n_{jt} and capital $\tilde{K}_{jt}/\tilde{N}_{jt} = k_{jt}$ are substitutable if the operation cost does not depend on capital so much; that is $\xi_j < \theta_j$. Furthermore, other things being equal, an increase in the normalized establishment-level productivity A_{jt}/ϕ_j allows the expansion of variety; that is it weakens the trade-off between the love of variety and increasing returns.

3.3. Determination of City Size

Given the result in the first step, the optimization problem in the second step is written as follows:

$$\max_{\mu_{jt}} \Big\{ \mu_{jt}(\tilde{Y}_{jt} - b\tilde{N}_{jt}^{\tilde{\gamma}}) = \Psi_j (A_{jt} K_{jt}^{\theta_j} N_{jt}^{1-\theta_j})^{\frac{1}{1-\xi_j}\left(\frac{\sigma}{\sigma-1}-\xi_j\right)} \times \left(\phi_j K_{jt}^{\xi_j}\right)^{-\frac{1}{(1-\xi_j)(\sigma-1)}} \mu_{jt}^{-\frac{1}{\sigma-1}} - bN_{jt}^{\tilde{\gamma}} \mu_{jt}^{1-\tilde{\gamma}} \Big\}, \quad (27)$$

where

$$\Psi_j \equiv \frac{(\sigma-1)\left(\frac{\sigma}{\sigma-1} - \xi_j\right) - 1}{\left[(\sigma-1)\left(\frac{\sigma}{\sigma-1} - \xi_j\right)\right]^{\frac{1}{1-\xi_j}\left(\frac{\sigma}{\sigma-1}-\xi_j\right)}}.$$

In determining the net output $\mu_{jt}(\tilde{Y}_{jt} - b\tilde{N}_{jt}^{\tilde{\gamma}})$, the social planner faces a trade-off between increasing returns to scale at the city level and economization of commuting cost.

To avoid the meaningless case, we assume that the resulting net output is positive. This requires us to assume the following:

Assumption 1. *$\tilde{\gamma} > \frac{\sigma}{\sigma-1}$ or, equivalently, $\gamma > \frac{2}{\sigma-1}$ such that the net output $\mu_{jt}(\tilde{Y}_{jt} - b\tilde{N}_{jt}^{\gamma})$ is positive.*

Then, as in Rossi-Hansberg and Wright (2007b), we obtain the aggregate constant returns to scale:

Proposition 2. *The industry technology exhibits constant returns to scale at the aggregate level:*

$$\mu_{jt}(\tilde{Y}_{jt} - b\tilde{N}_{jt}^{\tilde{\gamma}}) = \tilde{A}_{jt} K_{jt}^{\tilde{\theta}_j} N_{jt}^{1-\tilde{\theta}_j}, \quad (28)$$

where the TFP $\tilde{A}_{jt}$ and effective share parameter $\tilde{\theta}_j$ are given by

$$\begin{aligned} \tilde{A}_{jt} &\equiv \frac{[(\sigma-1)(\tilde{\gamma}-1)-1]b}{[(\sigma-1)(\tilde{\gamma}-1)b]^{\frac{\tilde{\gamma}-1}{\tilde{\gamma}-\frac{\sigma}{\sigma-1}}}} \times \left\{ \frac{(\sigma-1)\left(\frac{\sigma}{\sigma-1}-\xi_j\right)-1}{\left[(\sigma-1)\left(\frac{\sigma}{\sigma-1}-\xi_j\right)\right]^{\frac{1}{1-\xi_j}\left(\frac{\sigma}{\sigma-1}-\xi_j\right)}} \right\}^{\frac{\tilde{\gamma}-1}{\tilde{\gamma}-\frac{\sigma}{\sigma-1}}} \\ &\quad \times A_{jt}^{\frac{1}{1-\xi_j}\left(\frac{\sigma}{\sigma-1}-\xi_j\right)\frac{\tilde{\gamma}-1}{\tilde{\gamma}-\frac{\sigma}{\sigma-1}}} \phi_j^{-\frac{1}{(1-\xi_j)(\sigma-1)}\frac{\tilde{\gamma}-1}{\tilde{\gamma}-\frac{\sigma}{\sigma-1}}}, \quad (29) \\ \tilde{\theta}_j &\equiv \frac{\tilde{\gamma}-1}{\tilde{\gamma}-\frac{\sigma}{\sigma-1}} \frac{1}{1-\xi_j} \left[\theta_j \left(\frac{\sigma}{\sigma-1} - \xi_j \right) - \frac{\xi_j}{\sigma-1} \right]. \end{aligned}$$

The endogenous determination of the number μ_{jt} of cities results in balancing the increasing returns at the city level and economization of the commuting cost, leading to the constant returns to scale at the macro level.

In the following, we focus on the natural case:

Assumption 2. *The gross domestic product of any industry is an increasing function of capital and labor; that is $\tilde{\theta}_j \in (0, 1)$ for all j.*

Then, the city size $\tilde{N}_{jt}$ is given by

$$\tilde{N}_{jt} = \frac{1}{\{[(\sigma-1)(\tilde{\gamma}-1)-1]b\}^{\frac{1}{\tilde{\gamma}-1}}} \tilde{A}_{jt}^{\frac{1}{(1-\tilde{\theta}_j)(\tilde{\gamma}-1)}} (k_{jt}^e)^{\frac{\tilde{\theta}_j}{\tilde{\gamma}-1}}, \quad (30)$$

where $k_{jt}^e = K_{jt}/(\tilde{A}_{jt}^{\frac{1}{1-\tilde{\theta}_j}} N_{jt})$ is the capital stock per efficiency unit of labor. This implies that an increase in productivity $\tilde{A}_{jt}$ or capital k_{jt}^e raises workers' productivity in a given city. This allows an expansion of a city even if the commuting cost increases as the city size becomes larger.

3.4. Size Dynamics

Consolidating the above results of the first two steps and the original problem gives the following reduced problem, which is now expressed as the Bellman equation:

$$\begin{aligned} V(\boldsymbol{S}) &= \max_{\{C_j, I_j, N_j\}_{j=1}^J} \left\{ \sum_{j=1}^J \alpha_j \ln\left(\frac{C_j}{N}\right) + \tilde{\beta} E\left[V(\boldsymbol{S}')|\boldsymbol{S}\right] \right\} \\ & s.t. \\ & C_j + I_j = \tilde{A}_j K_j^{\tilde{\theta}_j} N_j^{1-\tilde{\theta}_j}, \\ & \sum_{j=1}^J N_j = N, \\ & K_j' = K_j^{\omega_j} I_j^{1-\omega_j}, \\ & \tilde{A}_j' = \left[e^{z_j' - z_j}(1+g_A)\right]^{\frac{1}{1-\xi_j}\left(\frac{\sigma}{\sigma-1}-\xi_j\right)\frac{\tilde{\gamma}-1}{\tilde{\gamma}-\frac{\sigma}{\sigma-1}}} \tilde{A}_j, \\ & z_j' = \rho_j z_j + \varepsilon_j \end{aligned}$$

where $\boldsymbol{S} = (\{K_j, \tilde{A}_j, z_j\}_{j=1}^J, N)$, the vector of state variables, and $\tilde{\beta} = \beta(1 + g_N)$, which is assumed to be less than one. We omit the time subscript t, and the prime denotes the next period. The law of motion for $\tilde{A}_j$ is obtained by consolidating (8) and (29).

This is nothing but a neoclassical optimal growth model with multiple industries. The standard argument of the dynamic programming tells us that the optimal solution is characterized by constant shares:

$$\begin{aligned} N_j &= s_j^n N, \\ I_j &= s_j^I \tilde{A}_j K_j^{\tilde{\theta}_j} N_j^{1-\tilde{\theta}_j}, \end{aligned} \quad (31)$$

where

$$\begin{aligned} s_j^n &\equiv \frac{(1-\tilde{\theta}_j)[\alpha_j + \tilde{\beta}(1-\omega_j)D_j^K]}{\sum_{j=1}^J (1-\tilde{\theta}_j)[\alpha_j + \tilde{\beta}(1-\omega_j)D_j^K]} \in (0,1), \\ s_j^I &\equiv \frac{\tilde{\beta}(1-\omega_j)D_j^K}{\alpha_j + \tilde{\beta}(1-\omega_j)D_j^K} \in (0,1), \\ D_j^K &\equiv \frac{\alpha_j \tilde{\theta}_j}{1 - \tilde{\beta}[\omega_j + (1-\omega_j)\tilde{\theta}_j]} > 0. \end{aligned}$$

Except for the specification of the law of motion for the capital stock, this is equivalent to the standard Solow growth model.

In what follows, we first derive the law of motion for the capital stock per efficiency unit of labor k^e_{jt}. Then, we derive the laws of motion both of the population $\tilde{N}_{jt}$ of a city and the number n_{jt} of employees per establishment by noting that these two variables are given as functions of k^e_{jt}.

3.4.1. Capital Stock

The law of motion for k^e_{jt} is computed by substituting the optimal investment policy function (31) into the law of motion for the aggregate capital stock K_{jt}:

$$\ln\left(\frac{k^e_{j,t+1}}{k^e_{jt}}\right) = (1-\omega_j)(1-\tilde{\theta}_j)[\ln(k^e_{j,ss}) - \ln(k^e_{jt})] - \frac{z_{j,t+1} - z_{jt}}{\xi_j - \theta_j}, \tag{32}$$

where $k^e_{j,ss}$ denotes the deterministic steady-state level of k^e_{jt} given by

$$k^e_{j,ss} = \frac{(s^I_j)^{1-\theta_j}}{[(1+g_y)(1+g_N)]^{\frac{1}{(1-\omega_j)(1-\tilde{\theta}_j)}}}.$$

g_y is the growth rate of $\tilde{A}_{jt}^{\frac{1}{1-\tilde{\theta}_j}}$, which corresponds to the growth rate of per capita income in a balanced growth path. In a deterministic case, that is $z_{jt} = 0$ for all j and t, $\{k^e_{jt}\}_{t=0}^\infty$ converges monotonically to $k^e_{j,ss}$ for any initial condition $k^e_{j,0}$. This is because the law of motion (32) exhibits a "mean reversion" property in the sense that the growth rate $k^e_{j,t+1}/k^e_{jt}$ is decreasing in the size k^e_{jt}.

3.4.2. City Size

The law of motion for the size $\tilde{N}_{jt}$ of a city is computed from (8), (29) and (30):

$$\ln\left(\frac{\tilde{N}_{j,t+1}}{\tilde{N}_{jt}}\right) = \frac{\tilde{\theta}_j}{\tilde{\gamma}-1}\ln\left(\frac{k^e_{j,t+1}}{k^e_{jt}}\right) + \frac{1}{(1-\tilde{\theta}_j)(1-\xi_j)}\frac{\frac{\sigma}{\sigma-1}-\xi_j}{\tilde{\gamma}-\frac{\sigma}{\sigma-1}} \times \left[z_{j,t+1} - z_{jt} + \ln(1+g_A)\right].$$

Note that the dynamics is consistent with the parallel growth mentioned by Eaton and Eckstein (1997). To see this, focus on the deterministic case, where $z_{jt} = 0$ for all t and j. Then, the steady-state growth rate is given by

$$\ln\left(\frac{\tilde{N}_{j,t+1}}{\tilde{N}_{jt}}\right) = \frac{1}{(1-\tilde{\theta}_j)(1-\xi_j)}\frac{\frac{\sigma}{\sigma-1}-\xi_j}{\tilde{\gamma}-\frac{\sigma}{\sigma-1}}\ln(1+g_A).$$

Since $\xi_j < 1$ and $\tilde{\gamma} > \sigma/(\sigma-1)$ by assumption, the coefficient of the right hand side is positive. Thus, cities grow if the productivity grows; that is $g_A > 0$. In addition, if parameters are symmetric, the growth rates become the same across cities.

3.4.3. Establishment Size

Before deriving the law of motion for the establishment size, we first restrict attention to the case where the following assumption holds:

Assumption 3. *In a deterministic steady state, the establishment size of any industry is constant.*

Eleven years data of the establishment size from SUSB reveals that no clear trend in the average establishment size can be seen for each three-digit industry except for manufacturing industries. We neglect the decreasing trend observed in manufacturing industries. This is because the current closed economy model cannot take into account of international trade factors, which might be crucial for explaining such a decreasing trend.

Then, the following result gives the formulation which gives the establishment size as well as a restriction to the relationship between the curvature parameter ξ_j of the operation cost and the importance θ_j of capital when we consider the economy with long-run growth:[13]

Proposition 3. *Assume that Assumption 3 holds. Then, (i) the long-run growth rate of per capita income is given by*

$$1 + g_y = (1+g_A)^{\frac{1}{\xi_j - \theta_j}}; \tag{33}$$

(ii) parameters must satisfy

$$\frac{\xi_j - \theta_j}{(1-\tilde{\theta}_j)(1-\xi_j)}\left(\frac{\sigma}{\sigma-1} - \xi_j\right)\frac{\tilde{\gamma}-1}{\tilde{\gamma}-\frac{\sigma}{\sigma-1}} = 1; \tag{34}$$

and (iii) the establishment size is given by

$$n_{jt} = \Upsilon_j \phi_j^{\frac{\sigma-1}{(1-\xi_j)\sigma+\xi_j}}(k^e_{jt})^{\frac{\xi_j-\theta_j}{1-\xi_j}}, \tag{35}$$

where

$$\Upsilon_j \equiv \left\{\frac{[(\sigma-1)(\tilde{\gamma}-1)-1]b}{[(\sigma-1)(\tilde{\gamma}-1)b]^{\frac{\tilde{\gamma}-1}{\tilde{\gamma}-\frac{\sigma}{\sigma-1}}}}\right\}^{\frac{\xi_j-\theta_j}{(1-\tilde{\theta}_j)(1-\xi_j)}} \times\left[(\sigma-1)\left(\frac{\sigma}{\sigma-1}-\xi_j\right)-1\right]^{\frac{1}{\frac{\sigma}{\sigma-1}-\xi_j}}.$$

Note that, under Assumption 3 we must assume that $\xi_j > \theta_j$ in order to allow the economy exhibit the long-run growth of income per capita. Since the establishment size n_{jt} is decreasing in productivity A_{jt} according to (26), the model requires the complementarity between capital and labor, that is $\xi_j > \theta_j$, in order to make n_{jt} constant over time.

The implied law of motion for the establishment is as follows:

$$\ln\left(\frac{n_{j,t+1}}{n_{jt}}\right) = \frac{\xi_j - \theta_j}{1-\xi_j}\ln\left(\frac{k^e_{j,t+1}}{k^e_{jt}}\right).$$

[13]For the first result, we use (26) and (29). For the second result, we use the definition of g_y and (29).

3.5. Size Distributions of Cities

Finally, we can show that the size distribution of cities exhibits concavity in the long run; that is, compared with Zipf's law, we observe the underrepresentation of both small and very large cities. This is what we observe in the data, which is mentioned as a robust systematic deviation from the Pareto distribution of coefficient one (See Gabaix and Ioannides (2004)).

The key mechanism is the mean reversion of the capital stock per efficiency unit of labor k^e_{jt}. Since larger (smaller) cities are associated with larger (smaller) capital stock, the mean reversion implies that they grow at rates lower (higher) than the average.[14] In a special case, where the mean reversion does not work, the size distribution of cities obeys the Pareto distribution with coefficient one.

The following proposition summarizes these discussions:[15]

Proposition 4.

1. *(i) Consider a economy with no storage technology, where there is no capital, that is $\theta_j = 0$; the operating cost does not depend on capital, that is $\xi_j = 0$; the shift factor ϕ_j grows at the same rate as the productivity A_{jt}; and the technology shock follows a random walk $\rho_j = 1$. Further, assume that the number J of industries is sufficiently large. Then the growth process of the city size exhibits Gibrat's law. (ii) In addition, if city sizes are bounded below by $f > 0$, then the long-run distribution of city sizes obeys Zipf's law as $f \to 0$.*
2. *If productivities $\{A_{jt}\}$ are uniformly bounded; and if the number J of industries is sufficiently large, there exists a unique invariant distribution of city sizes with underrepresentation of both small and very large cities than the Pareto distribution with coefficient one.*

4. Quantitative Analysis

In this section, we first investigate the extent to which the model can reproduce the observed joint distribution of city and average establishment sizes as well as the size distribution of cities under empirically plausible values of parameters. In order to do this, we calibrate the model to match some empirical data on growth, industries, and cities pertaining to the United States. Our result shows that this kind of model can reproduce both distributions except for generating variation of the establishment size more volatile than data.

Then, we conduct two counterfactual simulations in order to examine the dependence of the size distribution of cities on the industrial property, which is a crucial factor in determining the size distribution of establishments. In the first case, we assume that industries are symmetric. In the second case, we assume that the distribution of (ϕ_j, A_{j0}) is uniform in a certain sense.

We confirm that the size distribution of cities depends on the distribution of (ϕ_j, A_{j0}) in general. However, there is a range of distributions of (ϕ_j, A_{j0}) that variations of (ϕ_j, A_{j0}) does not change the size distribution of cities significantly. Especially for distributions of (ϕ_j, A_{j0}) close to the calibrated one, the size distribution of cities is determined almost independently of the distribution of (ϕ_j, A_{j0}).

However, when it comes to the case of the joint distribution of city and average establishment sizes, the distribution of (ϕ_j, A_{j0}) has a crucial role. The results show that self selection within industries and a lower skewed distribution of (ϕ_j, A_{j0}) is necessary in order to generate the empirically observed positive correlation between the size of a city and the average establishment size in the city.

In what follows, we first describe the calibration of the model. Then, the results of the counterfactual simulations are presented.

4.1. Calibration

4.1.1. Strategy

In order to focus on heterogeneity of the initial productivity A_{j0} and the shift parameter ϕ_j of the operating cost, we assume that values of the rest of parameters are the same across industries:

$$\alpha_j = \alpha, \quad \theta_j = \theta, \quad \omega_j = \omega, \quad \xi_j = \xi, \quad \rho_j = \rho, \quad \sigma_{\varepsilon_j} = \sigma_\varepsilon.$$

Note that we allow each industry to have a different sample path $\{z_{jt}\}_t$ of productivity shocks.

Thus, given the number J of industries, there are $2J + 12$ parameters, each of which is categorized into those calibrated independently without simulating the model and those calibrated jointly by simulating the model. The former consist of (i) the discount factor β, (ii) expenditure parameter α, (iii) population growth rate g_N, (iv) persistence ρ of productivity shock, (v) standard deviation σ_ε of the innovation, (vi) importance θ of capital, (vii) persistence parameter ω in the law of motion for capital, (viii) curvature parameter γ of the commuting cost, and (ix) shift parameter τ of the commuting cost. The rest $2J + 3$ parameters are (i) the elasticity σ of substitution between intermediate goods, (ii) curvature parameter ξ of the operating cost, (iii) long-run growth rate g_A of productivity, (iv) shift parameters $\{\phi_j\}_{j=1}^J$ of the operating cost, and (v) initial productivities $\{A_{0j}\}_{j=1}^J$.

The choice of J depends on the industry. In this paper, we focus on the three-digit industries according to the 2002 North American Industrial Classification System (NAICS). The available number of industries is 53, and thus, we set $J = 53$.[16] Here,

[14] (30) shows that the city size $\tilde{N}_{jt}$ is increasing in the capital stock per efficiency unit of labor $\tilde{k}_{jt}$. In addition, the law of motion of k^e_{jt}, given by (32), implies that the growth rate is likely negative (positive) when k^e_{jt} is above (below) its steady-state level $k^e_{j,ss}$. These two results together imply the mean reversion of cities.

[15] Since the approach in proving the proposition is the same as Rossi-Hansberg and Wright (2007b), we omit the proof. We can show that the same result applies to the case of the size distribution of establishments. However we omit it from the proposition since we do not make use of it in the quantitative analysis.

[16] Since we need to calibrate $\{A_{j0}, \phi_j\}_{j=1}^J$, we must restrict our attention to some industrial classification for which data necessary for our calibration are available.

β	α	g_N	ρ	σ_ε	θ
0.939	1/53	0.012	0.76	0.07	0.3

ω	γ	τ	σ	ξ	g_A
0.9	1	10	4.35	0.403	0.002

Table 1: Parameters

we exclude three-digit industries in "Agriculture, forestry, fishing, and hunting," "Mining," "Utilities," and "Construction" in order to focus on urban economic activities.

4.1.2. Independently Calibrated Parameters

Values of parameters except for $\{\phi_j, A_{0j}\}_{j=1}^J$ are shown in Table 1. The value of β implies that the average annual real rate of return is about 5%. $\alpha = 1/J$ since expenciture shares are the same across industries. g_N is set so that the annual population growth rate is equal to 1.2%. ρ and σ_ε are consistent with the estimate of Horvath (2000). θ is slightly lower than one-third since capital is included in the operating cost. ω and γ is the same as in Rossi-Hansberg and Wright (2007b). The value of τ is arbitrary since it does not affect *relative* size of cities.

4.1.3. Jointly Calibrated Parameters

Jointly calibrated parameters are obtained by following the steps:

1. Choose a value of σ.
2. Compute ξ which satisfies (34). If $\tilde{\theta} \in (0, 1)$, proceed to the next step. Otherwise, go back to step 1.
3. Compute g_A from (33). where the annual growth rate g_y of per capita income is set to 2%.
4. Compute $\{\tilde{A}_{j0}\}_{j=1}^J$ by using (28) and data on GDP, capital stock, and employments.
5. Compute $\{\phi_j\}_{j=1}^J$ by substituting data on the number of employees per establishment and the value of k_{jt}^e, implied by step 4, into (35) and solving the result for ϕ_j.
6. Compute $\{A_{0j}\}_{j=1}^J$ by substituting the previous steps into (29).
7. Regress the natural logarithm of A_{0j} on that of ϕ_j and reset A_{0j} to the fitted value of the regression.
8. Simulate the model in order to obtain the mean of the size distribution of cities and compare the result with the actual size distribution of 597 MSAs in U.S. in 2008. If the difference between the two is sufficiently close, stop. Otherwise, go back to step 1.

In step 3, we set the annual growth rate g_y of per capita income to 2%. In step 4, we use the GDP-by-industry accounts and Fixed Assets Accounts of the Bureau of Economic Analysis (BEA) in 2007 for GDP and capital stock, respectively. The number of employees is based on SUSB in 2007. In step 5, we use data on the number of employees per establishment in SUSB in 2007. Figure 2 depicts the relationship between ϕ_j and A_{0j} which are calibrated in step 5, 6 and 7, respectively. It suggests that we can approximate the relationship by a log-linear

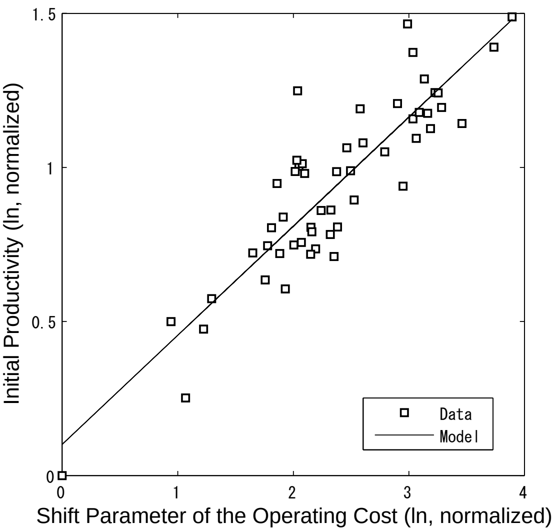

Figure 2: The Shift Parameter of the Operating Cost and the Initial Productivity

form. In step 8, the mean of the size distribution of cities is obtained by conducting 1,000 Monte Carlo simulations. In each case, we first compute the size distribution $\{\tilde{N}_{jt}, \mu_{jt}\}_{j=1}^J$ of cities by setting the time period to be 10,000 years. Then, computing the cumulative distribution function of city sizes, we generate 597 cities by random sampling with a linear interpolation.

The resulting relationship between the shift parameter ϕ_j of the operating cost and the TFP $\tilde{A}_{jt}$ is slightly positive. Since, for a fixed level of productivity A_{jt}, the TFP $\tilde{A}_{jt}$ is decreasing in ϕ_{jt}, self selection within industries, which is exogenously assumed and depicted in Figure 2, is sufficient in order to dominate this negative direct effect of ϕ_j on $\tilde{A}_{jt}$.

Figure 3 shows size distributions of cities in both the model and data as well as the 90 percent confidence interval computed by the Monte Carlo simulation. The size distribution of cities observed in the data not only is included in the 90 percent confidence interval but also differs little from the mean of the size distribution of cities in the model.

Note that the calibrated value of σ, 4.35, belongs to the empirically plausible range of σ (e.g. Broda and Weinstein, 2006). From these results, we infer that this kind of model, that is model with monopolistic competition and endogenous city creation, is a candidate model, which can reproduce the observed size distribution of cities.

4.2. Joint Distribution of City and Average Establishment Sizes

We can also evaluate the model by adding the joint distribution of city and average establishment sizes as a new restriction to economic modeling. Thus, we compare the simulation result with the data from this point of view. In simulation, we apply the same method as in the previous calibration except that we conduct 100 Monte Carlo simulations.

Figure 4 shows the result thus obtained. The actual joint distribution is included in the possible range of the simulated joint distribution.

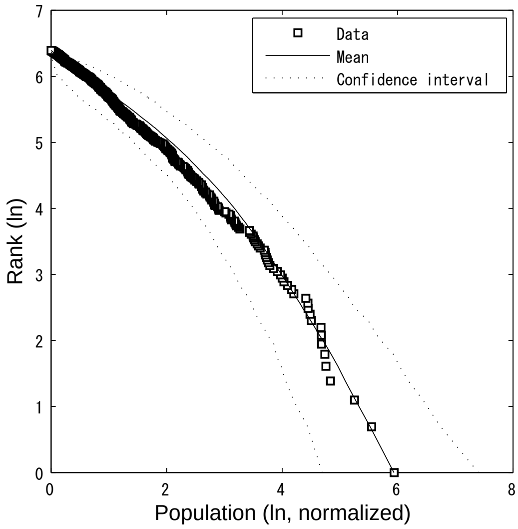

Figure 3: Size Distribution of Cities

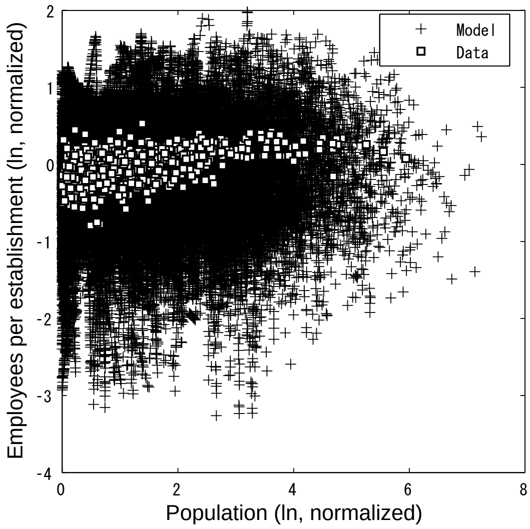

Figure 4: Joint Distribution of City and Average Establishment Sizes

However, we note that there is excessive variation in the *relative* size of establishments in the model. This could be the result of three properties of the model:[17] (i) There is no adjustment cost of labor. If adjusting labor is costly, the variation in the establishment size would be smaller; (ii) There is no human capital. If establishments could use human capital as an additional input, the variation in the number of employees per establishment would be absorbed to some extent by the variation in human capital per establishments; and (iii) A city hosts only one industry. If a city hosts multiple industries and if there is hierarchy of cities as mentioned by Mori et al. (2008), the shift parameter of the operating cost specific to the city is the weighted average of $\{\phi_j\}_{j=1}^J$. Then, the variation in labor would be eliminated to some extent since the variation in the shift parameter would decrease.

4.3. Implications of Industrial Properties

Previous results suggest that our model can be used as a benchmark in the study of the determination of the joint distribution of city and average establishment sizes. However, we have left the question of the interdependence between the size distribution of cities and the size distribution of establishments.

We thus finally conduct two counterfactual simulations in order to answer this question. In the first case, we assume that industries are symmetric in the sense that parameters including $\{\phi_j, A_{j0}\}_{j=1}^J$ are the same across industries. In the second case, we assume that the distribution of (ϕ_j, A_{j0}) is uniform in the sense that ϕ_j is uniformly distributed on the interval $[\min\{\phi_j\}, \max\{\phi_j\}]$; and that A_{j0} is the fitted value of the regression with the same regression coefficient as in the previous subsections. Because of limitations of the model, we consider only the dependence of the size distribution of cities on the distribution of (ϕ_j, A_{j0}), which is a crucial factor in determining the size distribution of establishments.

4.3.1. Symmetric Industries

Figure 6 and 7 show the size distribution of cities and the joint distribution of city and average establishment sizes under the assumption that industries are symmetric, respectively.[18] The former shows that the size distribution of cities differs little from the benchmark case. Stated differently, the change from the counterfactual distribution of ϕ_j to the benchmark one, depicted in Figure 5, has negligible impact on the size distribution of cities.

The latter shows that the correlation between the natural logarithm of city sizes and that of average establishment sizes is slightly negative. Comparison between the size determination of cities, (30), and that of establishments, (35), makes clear the mechanism behind this result. The difference is that the city size $\tilde{N}_{jt}$ depends not only the capital stock per efficiency unit of labor $\tilde{k}^e_{jt}$ but also the TFP $\tilde{A}_{jt}^{\frac{1}{1-\theta_j}}$ while the establishment size n_{jt} depends on $\tilde{k}^e_{jt}$ only. Since, other things being equal, $\tilde{k}^e_{jt}$ is decreasing in $\tilde{A}_{jt}^{\frac{1}{1-\theta_j}}$ by definition, the negative correlation between $\tilde{N}_{jt}$ and n_{jt} can emerge.[19]

Comparing this result with the benchmark case, we can conclude that we need variation of ϕ_js and self selection within each industry in order to generate the positive correlation between city and establishment sizes. The variation of ϕ_js together with self selection generates additional variation of the

[17]Relaxing these assumptions is one direction of future research.

[18]In this case, values of ϕ_j and A_{0j} does not matter to relative sizes.

[19]If industries are symmetric, ϕ_js are equal across industries. Thus, the positive correlation between $\tilde{N}_{jt}$ and n_{jt} through the positive relation between the TFP $\tilde{A}_{jt}$ and ϕ_j is eliminated.

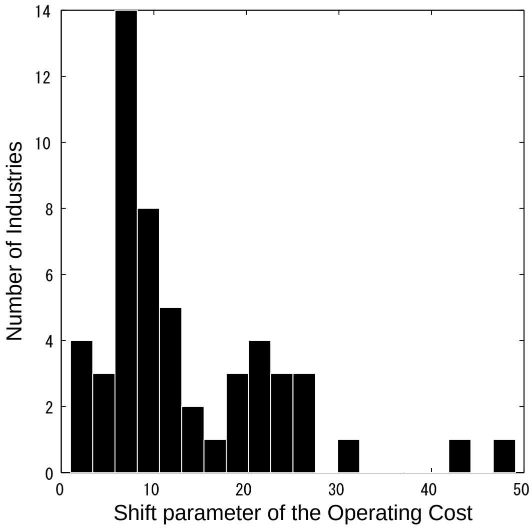

Figure 5: Histogram of $\{\phi_j\}$: The Benchmark Case

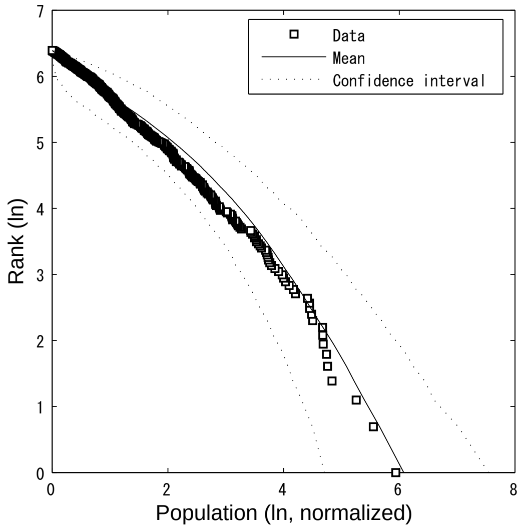

Figure 6: Size Distribution of Cities: The Case of Symmetric Industries

size of cities, which is systematic in a sense that these generate a positive correlation between the city size $\tilde{N}_{jt}$ and the establishment size n_{jt}. In addition, this effect dominates the effect of the relationship between the TFP and the capital stock per efficiency unit of labor.

4.3.2. Uniform Distribution of (ϕ_j, A_{j0})

Figure 8 shows the size distribution of cities under the assumption that the distribution of (ϕ_j, A_{0j}) is uniform as defined previously. The size distribution of cities drastically changes from the benchmark case. Especially, the confidence interval expands. This is because, compared with the distribution of ϕ_j in the benchmark case depicted in Figure 5, the uniform distribution of ϕ_j results in the greater variation in the level of TFPs $\tilde{A}_{jt}$. (30) then suggests that the variation of $\tilde{N}_{jt}$ becomes larger.[20] The mean of the size distribution of cities shows that the share of cities in the middle range increases compared with the benchmark case, and this is what is predicted.

5. Conclusion

In this paper, we build a dynamic general equilibrium growth model with multiple industries, in which the joint distribution of city and average establishment sizes is determined endogenously. Monopolistic competition in the intermediate goods sector of industries plays an important role in determining the size distribution of establishments. The establishment-level increasing returns then leads to the scale economy at the city level. The size distribution of cities is determined by the tradeoff between this and the commuting cost.

[20]Note that the dynamics of the capital stock per efficiency unit of labor k^e_{jt} is the same across all cases.

The quantitative analysis suggests that this kind of model is promising in understanding the joint distribution of city and average establishment sizes. The model, calibrated to the U.S. economy, can reproduce both the observed size distribution of cities and the joint distribution of city and average establishment sizes except for generating variation of the establishment size more volatile than data.

Counterfactual simulations show that the size distribution of cities depends on the industrial properties such as the distribution of operating costs of production and productivities in general. Depending on the distribution, the city size distribution is almost independent of it.

It is also shown that variation in operating costs and self selection within industries are necessary in order to generate the positive correlation between the city and establishment sizes. Large industries, characterized by higher operating costs and thus higher average establishment sizes, have higher productivities thanks to self selection. TFPs of cities are determined in a way that such positive correlation between the operating cost and the productivity is not eliminated and results in the positive correlation between the operating cost and the TFP. Then, cities, which host larger industries, are able to expand.

There are several future research directions. Taking into account of hierarchy principle mentioned by Mori et al. (2008) is one of the most important extensions. This extension could not only reduce the excessive variation of the average size of establishments but also enrich the economic modeling of the joint distribution of city and average establishment sizes by adding the "Number-Average Size" rule, argued by Mori et al. (2008), as an additional restriction.

Endogenizing self selection within industries is also one of the most important extensions. In this paper, we imposed self selection exogenously. Thus, the quantitative analysis considered only the dependence of the size distribution of cities on the

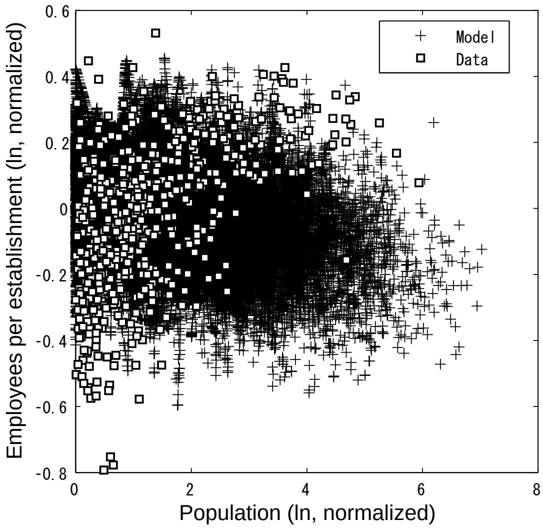

Figure 7: Joint Distribution of City and Average Establishment Sizes: The Case of Symmetric Industries

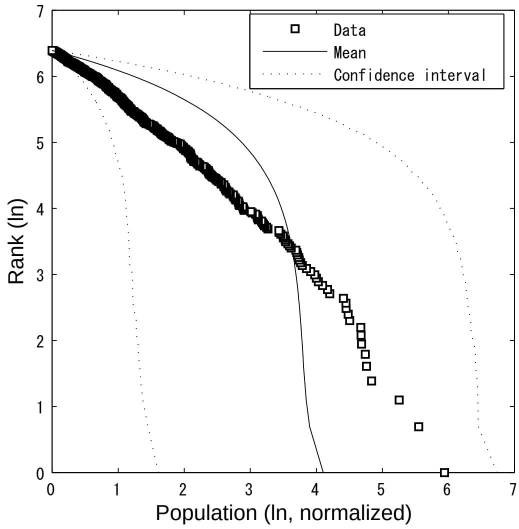

Figure 8: Size Distribution of Cities: The Case of Uniform Distribution of (ϕ_j, A_{j0})

industrial property. The converse causality works if we introduced endogenous self selection a la Hopenhayn (1992). This allows us to more understand the interdependence of the size distributions of cities and establishments.

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Appendix A.

The proof of Proposition 1 in the text proceeds as follows:

1. prove the uniqueness of the solution to the social planner's problem;
2. derive the first-order conditions (FOCs) for the social planner's problem;
3. derive the FOCs in the case of market equilibrium; and
4. show correspondence between FOCs for the social planner's problem and those for the market equilibrium.

Step1: Uniqueness of the Solution to the Social Planner's Problem

The social planner's problem is expressed as follows:

$$\max_{\{C_{jt},I_{jt},N_{jt}\}_{t=0}^{\infty}} E_0 \sum_{t=0}^{\infty} \beta^t \left[N_t \sum_{j=1}^{J} \alpha_j \ln\left(\frac{C_{jt}}{N_t}\right)\right] \tag{A.1}$$

s.t.

$$C_{jt} + I_{jt} \leq \mu_{jt}(\tilde{Y}_{jt} - b\tilde{N}_{jt}^{\tilde{\gamma}}), \tag{A.2}$$

$$\tilde{Y}_{jt} = \tilde{M}_{jt}^{\frac{\sigma}{\sigma-1}} \left(\frac{A_{jt}\tilde{K}_{jt}^{\theta_j}\tilde{N}_{jt}^{1-\theta_j}}{\tilde{M}_{jt}} - \phi_j \frac{\tilde{K}_{jt}^{\xi_j}}{\tilde{M}_{jt}^{\xi_j}} \right), \tag{A.3}$$

$$K_{jt} \geq \mu_{jt}\tilde{K}_{jt}, \tag{A.4}$$

$$N_{jt} \geq \mu_{jt}\tilde{N}_{jt}, \tag{A.5}$$

$$\sum_{j=1}^{J} N_{jt} \leq N_t, \tag{A.6}$$

$$K_{j,t+1} \leq K_{jt}^{\omega_j} I_{jt}^{1-\omega_j}. \tag{A.7}$$

As mentioned in the text, the social planner solves the problem in three steps, each having a maximization problem. The uniqueness of the solution is proved by showing that there exists a unique solution to the problem in each step.

First, the planner maximizes the output $\tilde{Y}_{jt}$ of intermediate goods in each city by changing the number $\tilde{M}_{jt}$ of varieties. For positive values of the capital stock $\tilde{K}_{jt}$ and the labor input $\tilde{N}_{jt}$ of each city, there exists a unique positive level of $\tilde{M}_{jt}$, which maximizes $\tilde{Y}_{jt}$, if $\xi_j < 1$.

Second, the planner maximizes the net output $\mu_{jt}(\tilde{Y}_{jt} - b\tilde{N}_{jt}^{\tilde{\gamma}})$ of each industry by changing the number μ_{jt} of cities. Since $\mu_{jt}(\tilde{Y}_{jt} - b\tilde{N}_{jt}^{\tilde{\gamma}})$ has the form given by (27), for positive values of K_{jt} and N_{jt}, there is a unique positive value of μ_{jt} which maximizes $\mu_{jt}(\tilde{Y}_{jt} - b\tilde{N}_{jt}^{\tilde{\gamma}})$ if Assumption 1 in the text holds, i.e.,

$\tilde{\gamma} > \sigma/(\sigma-1)$.

Finally, the social planner solves the reduced problem, which is obtained by substituting the results of the above two steps into the original problem:

$$\max_{\{C_{jt},I_{jt},N_{jt}\}_{t=0}^{\infty}} E_0 \sum_{t=0}^{\infty} \beta^t \left[N_t \sum_{j=1}^{J} \alpha_j \ln\left(\frac{C_{jt}}{N_t}\right)\right]$$
$$s.t.$$
$$C_{jt} + I_{jt} \le \tilde{A}_{jt} K_{jt}^{\tilde{\theta}_j} N_{jt}^{1-\tilde{\theta}_j},$$
$$\sum_{j=1}^{J} N_{jt} \le N_t,$$
$$K_{j,t+1} \le K_{jt}^{\omega_j} I_{jt}^{1-\omega_j}.$$

Since this is a convex problem, there exists a unique solution.

Step2: FOCs for the Social Planner's Problem

For later use, we derive the FOCs for the original problem: (A.1) - (A.7). They are expressed as follows:

$$\tilde{\lambda}_{jt}^{RF} = \beta^t N_t \alpha_j \frac{1}{C_{jt}}, \tag{A.8}$$

$$\tilde{\lambda}_{jt}^{N} = \tilde{\lambda}_{t}^{N}, \tag{A.9}$$

$$\tilde{\lambda}_{jt}^{RF} = (1-\omega_j)\tilde{\lambda}_{jt}^{DK}\left(\frac{K_{jt}}{I_{jt}}\right)^{\omega_j}, \tag{A.10}$$

$$\tilde{\lambda}_{jt}^{DK} = E_t\left[\tilde{\lambda}_{j,t+1}^{K} + \omega_j \tilde{\lambda}_{j,t+1}^{DK}\left(\frac{I_{j,t+1}}{K_{j,t+1}}\right)^{1-\omega_j}\right],$$

$$\frac{1}{\sigma-1}\tilde{M}_{jt}^{\frac{1}{\sigma-1}-1} A_{jt}\tilde{K}_{jt}^{\theta_j}\tilde{N}_{jt}^{1-\theta_j} = \left(\frac{\sigma}{\sigma-1}-\xi_j\right)\tilde{M}_{jt}^{\frac{\sigma}{\sigma-1}-\xi_j-1}\phi_j\tilde{K}_{jt}^{\xi_j}, \tag{A.11}$$

$$\tilde{\lambda}_{jt}^{K} = \tilde{\lambda}_{jt}^{RF}\left[\tilde{M}_{jt}^{\frac{1}{\sigma-1}}\theta_j A_{jt}\left(\frac{\tilde{N}_{jt}}{\tilde{K}_{jt}}\right)^{1-\theta_j} - \tilde{M}_{jt}^{\frac{\sigma}{\sigma-1}-\xi_j}\xi_j\phi_j\tilde{K}_{jt}^{\xi_j-1}\right], \tag{A.12}$$

$$\tilde{\lambda}_{jt}^{N} = \tilde{\lambda}_{jt}^{RF}\left[\tilde{M}_{jt}^{\frac{1}{\sigma-1}}(1-\theta_j)A_{jt}\left(\frac{\tilde{K}_{jt}}{\tilde{N}_{jt}}\right)^{\theta_j} - \tilde{\gamma} b\tilde{N}_{jt}^{\tilde{\gamma}-1}\right], \tag{A.13}$$

$$\tilde{\lambda}_{jt}^{K}\tilde{K}_{jt} + \tilde{\lambda}_{jt}^{N}\tilde{N}_{jt} = \tilde{\lambda}_{jt}^{RF}\left(\tilde{M}_{jt}^{\frac{1}{\sigma-1}}A_{jt}\tilde{K}_{jt}^{\theta_j}\tilde{N}_{jt}^{1-\theta_j} - \tilde{M}_{jt}^{\frac{\sigma}{\sigma-1}-\xi_j}\phi_j\tilde{K}_{jt}^{\xi_j} - b\tilde{N}_{jt}^{\tilde{\gamma}}\right), \tag{A.14}$$

where $\tilde{\lambda}_{jt}^{RF}$, $\tilde{\lambda}_{jt}^{K}$, $\tilde{\lambda}_{jt}^{N}$, $\tilde{\lambda}_{t}^{N}$ and $\tilde{\lambda}_{jt}^{DK}$ are the Lagrange multipliers corresponding to (A.2), (A.4), (A.5), (A.6) and (A.7), respectively. The multiplier corresponding to the gross output $\tilde{Y}_{jt}$ of the intermediate good does not appear since we eliminate $\tilde{Y}_{jt}$ by substituting (A.3) into (A.2).

We eliminate $\tilde{\lambda}_{jt}^{K}$ and $\tilde{\lambda}_{jt}^{N}$ from the FOCs for ease of the comparison discussed later. This requires three steps. First, substituting (A.9) into (A.13), we obtain

$$\tilde{\lambda}_{jt}^{RF}\left[\tilde{M}_{jt}^{\frac{1}{\sigma-1}}(1-\theta_j)A_{jt}\left(\frac{\tilde{K}_{jt}}{\tilde{N}_{jt}}\right)^{\theta_j} - \tilde{\gamma} b\tilde{N}_{jt}^{\tilde{\gamma}-1}\right] = \tilde{\lambda}_{t}^{N}. \tag{A.15}$$

Second, substituting (A.12) into (A.11), we obtain

$$\tilde{\lambda}_{jt}^{DK} = E_t\left\{\tilde{\lambda}_{j,t+1}^{RF}\left[\tilde{M}_{j,t+1}^{\frac{1}{\sigma-1}}\theta_j A_{j,t+1}\left(\frac{\tilde{N}_{j,t+1}}{\tilde{K}_{j,t+1}}\right)^{1-\theta_j} - \tilde{M}_{j,t+1}^{\frac{\sigma}{\sigma-1}-\xi_j}\xi_j\phi_j\tilde{K}_{j,t+1}^{\xi_j-1}\right] + \omega_j\tilde{\lambda}_{j,t+1}^{DK}\left(\frac{I_{j,t+1}}{K_{j,t+1}}\right)^{1-\omega_j}\right\}. \tag{A.16}$$

Finally, substituting (A.12) and (A.13) into (A.14), we obtain

$$(\tilde{\gamma}-1)b\tilde{N}_{jt}^{\tilde{\gamma}} = \tilde{M}_{jt}^{\frac{\sigma}{\sigma-1}-\xi_j}(1-\xi_j)\phi_j\tilde{K}_{jt}^{\xi_j}. \tag{A.17}$$

Step3: FOCs in the Case of a Market Equilibrium

Since the FOCs for problems of both final good firms and intermediate good firms are derived in the text, only those for developers' and households' problems are discussed here.

(i) Developer

To derive the FOCs for the developer's problem, it is convenient to express the developer's profit $TR_{jt} - T_{jt}^{n}\tilde{N}_{jt} - \frac{\tau_{jt}^{k} r_{jt}}{P_{jt}}\tilde{K}_{jt} - T_{jt}^{M}\tilde{M}_{jt}$ as a function of an appropriately chosen set of control variables. Thus, we first consider the set of control variables. Then, we derive the FOCs.

The set of control variables is the city-level capital stock $\tilde{K}_{jt}$, labor $\tilde{N}_{jt}$, and the number $\tilde{M}_{jt}$ of establishments. The planner can control these variables through subsidies $(\tau_{jt}^{k}, T_{jt}^{n}, T_{jt}^{M})$. Note that these are sufficient statistics for other control variables of the developer: the establishment-level capital stock k_{jt}, labor n_{jt} and the individual price level p_{jt}. k_{jt} and n_{jt} are determined by their definitions, i.e., $k_{jt} = \tilde{K}_{jt}/\tilde{M}_{jt}$ and $n_{jt} = \tilde{N}_{jt}/\tilde{M}_{jt}$. p_{jt} is determined by the optimal pricing rule of intermediate good firms. Note also that the social planner takes the aggregate price P_{jt}, factor prices (r_{jt}, w_{jt}) and the net income Ω_{jt} as given. This is because these are determined by competition between developers and free mobility of capital and labor.

The developer's profit is expressed as follows by using definitions, $k_{jt} = \tilde{K}_{jt}/\tilde{M}_{jt}$ and $n_{jt} = \tilde{N}_{jt}/\tilde{M}_{jt}$, (1), (2), (3), (6), (10), (11), (12), (13), and (14) in the text:

$$\tilde{M}_{jt}^{\frac{1}{\sigma-1}}A_{jt}\tilde{K}_{jt}^{\theta_j}\tilde{N}_{jt}^{1-\theta_j} - \phi_j\tilde{M}_{jt}^{\left(\frac{\sigma}{\sigma-1}-\xi_j\right)}\tilde{K}_{jt}^{\xi_j} - b\tilde{N}_{jt}^{\tilde{\gamma}} - \Omega_{jt}\tilde{N}_{jt} - \frac{r_{jt}}{P_{jt}}\tilde{K}_{jt}. \tag{A.18}$$

Given Ω_{jt} and r_{jt}/P_{jt}, this is a function of $(\tilde{K}_{jt}, \tilde{N}_{jt}, \tilde{M}_{jt})$.

Therefore, the FOCs are

$$\frac{r_{jt}}{P_{jt}} = \tilde{M}_{jt}^{\frac{1}{\sigma-1}}\theta_j A_{jt}\left(\frac{\tilde{N}_{jt}}{\tilde{K}_{jt}}\right)^{1-\theta_j} - \tilde{M}_{jt}^{\frac{\sigma}{\sigma-1}-\xi_j}\xi_j\phi_j\tilde{K}_{jt}^{\xi_j-1},$$

$$\Omega_{jt} = \tilde{M}_{jt}^{\frac{1}{\sigma-1}}(1-\theta_j)A_{jt}\left(\frac{\tilde{K}_{jt}}{\tilde{N}_{jt}}\right)^{\theta_j} - \tilde{\gamma} b\tilde{N}_{jt}^{\tilde{\gamma}-1}, \tag{A.19}$$

$$\frac{1}{\sigma-1}\tilde{M}_{jt}^{\frac{1}{\sigma-1}-1}A_{jt}\tilde{K}_{jt}^{\theta_j}\tilde{N}_{jt}^{1-\theta_j} = \left(\frac{\sigma}{\sigma-1}-\xi_j\right)\phi_j\tilde{M}_{jt}^{\frac{\sigma}{\sigma-1}-\xi_j-1}\tilde{K}_{jt}^{\xi_j}. \tag{A.20}$$

Substituting these FOCs into the objective function (A.18) gives the maximized profit:

$$(\tilde{\gamma}-1)b\tilde{N}_{jt}^{\tilde{\gamma}} - \tilde{M}_{jt}^{\frac{\sigma}{\sigma-1}-\xi_j}(1-\xi_j)\phi_j\tilde{K}_{jt}^{\xi_j},$$

which must be zero in equilibrium because of free entry and exit:

$$(\tilde{\gamma}-1)b\tilde{N}_{jt}^{\tilde{\gamma}} = \tilde{M}_{jt}^{\frac{\sigma}{\sigma-1}-\xi_j}(1-\xi_j)\phi_j\tilde{K}_{jt}^{\xi_j}.$$

(ii) Household

The household problem is expressed as follows:

$$\max_{\{C_{jt},I_{jt},N_{jt}\}_{j\in\{1,2,\cdots,J\},t\in\mathbb{R}_+}} E_0\sum_{t=0}^{\infty}\beta^t\left[N_t\sum_{j=1}^{J}\alpha_j\ln\left(\frac{C_{jt}}{N_t}\right)\right]$$

$s.t.$

$$\sum_{j=1}^{J}P_{jt}[C_{jt}+I_{jt}+(ACC_{jt}+AR_{jt})N_{jt}] \le \sum_{j=1}^{J}(w_{jt}N_{jt}+r_{jt}K_{jt}+P_{jt}T_{jt}^{n}N_{jt}), \tag{A.21}$$

$$\sum_{j=1}^{J}N_{jt}\le N_t, \tag{A.22}$$

$$K_{j,t+1}\le K_{jt}^{\omega_j}I_{jt}^{1-\omega_j}. \tag{A.23}$$

The household takes prices (P_{jt}, w_{jt}, r_{jt}), the average commuting cost ACC_{jt}, the average rent AR_{jt} and the transfer T_{jt}^{n} as given.

Therefore, the FOCs are

$$\lambda_t^{BC}P_{jt} = \beta^t N_t\alpha_j\frac{1}{C_{jt}}, \tag{A.24}$$

$$\lambda_t^{N} = \lambda_t^{BC}P_{jt}\left[\frac{w_{jt}}{P_{jt}}+T_{jt}^{n}-(ACC_{jt}+AR_{jt})\right], \tag{A.25}$$

$$\lambda_t^{BC}P_{jt} = (1-\omega_j)\lambda_{jt}^{DK}\left(\frac{K_{jt}}{I_{jt}}\right)^{\omega_j}, \tag{A.26}$$

$$\lambda_{jt}^{DK} = E_t\left[\lambda_{t+1}^{BC}P_{j,t+1}\frac{r_{j,t+1}}{P_{j,t+1}}+\omega_j\lambda_{j,t+1}^{DK}\left(\frac{I_{j,t+1}}{K_{j,t+1}}\right)^{1-\omega_j}\right],$$

where λ_t^{BC}, λ_t^{N} and λ_{jt}^{DK} are the Lagrange multipliers corresponding to (A.21), (A.22) and (A.23), respectively.

Step4: Correspondence between FOCs

In the following, it is shown that the FOCs for the social planner's problem is derived from the definition of a market equilibrium including the FOCs for agents' problems. This requires two steps: First, we relate the Lagrange multipliers as follows:

$$\tilde{\lambda}_{jt}^{RF} = \lambda_t^{BC}P_{jt}, \tag{A.27}$$

$$\tilde{\lambda}_{jt}^{DK} = \lambda_{jt}^{DK}, \tag{A.28}$$

$$\tilde{\lambda}_t^{N} = \lambda_t^{N}. \tag{A.29}$$

Second, we use these relationships to derive each of the FOCs for the planner's problem. Specifically, they consist of (A.8), (A.10), (A.11), (A.15), (A.16), and (A.17) as well as the resource constraints: (A.2) - (A.7) with equality. Since the derivation of the resource constraints is immediate, we discuss about only (A.8), (A.10), (A.11), (A.15), (A.16) and (A.17).

- (A.8) is obtained by substituting (A.27) into (A.24).
- (A.10) is obtained by substituting (A.27) and (A.28) into (A.26).
- (A.11) is equivalent to (A.20).
- (A.15) is obtained by substituting (14) in the text, (A.19) and (A.29) into (A.25).
- (A.16) is obtained by substituting (A.19), (A.27) and (A.28) into (A.27).
- (A.17) is equivalent to (A.21).

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