

# Sequential Estimators for Monitoring Autoregressive Processes with $p$ -th Order

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## Abstract

We consider sequential estimation for stationary  $p$ -th-order autoregressive (AR( $p$ )) processes under independent and identically distributed errors with zero mean and finite variance. Using the stopping time introduced by Lai and Siegmund (1983) for first-order autoregressive process (AR(1)), we investigate the joint asymptotic property of the stopping times, the sequential least estimator, and the estimator of the variance of independent errors for AR( $p$ ) process. Functional central limit theorem for nonlinear ergodic stationary processes plays important roles to get our main results with respect to the asymptotic properties of the stopping times, the sequential least square estimators, and the estimator of the variance of errors have the joint asymptotic normality. We derive the asymptotic normality of the stopping times. We found that the asymptotic variance of the stopping time with known variance is strictly greater than that of the stopping time with the variance estimator. Simulation studies are also conducted.

## Keywords

Statistical process monitoring; Time series; Observed Fisher information; Asymptotic normality of stopping time; Functional central limit theorem