Unit Root Tests in Sequentially Observed Autoregressive Processes

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Abstract

We consider unit root tests under monitoring sequentially observed autoregressive (AR) processes. We define stopping times based on the observed Fisher information and examine the asymptotic properties of unit root tests evaluated at the stopping times. We propose three unit root tests: a t-type test (T test), a test using the distribution of the stopping time (ST test), and a Bonferroni test (BON test). The asymptotic properties under alternative hypotheses of local-to-unity are considered by approximation via Orstein-Uhlenbeck process. The sequential autoregressive coefficient estimator is approximating to a Dambis, Dubins, and Schwarz (DDS) Brownian motion under unit root hypothesis and to a DDS Brownian motion with drift under a local-to-unity hypothesis. The asymptotic distribution of the stopping time is characterized by a Bessel process of dimension 3/2 under unit root hypothesis and also by a Bessel process of dimension 3/2 with drift under a local-to-unity hypothesis. T test turns out to be possessing local asymptotic normality (LAN). We implement Monte Carlo simulations and numerical computations to examine their small sample properties.

Keywords

Local-to-unity hypothesis; Stopping time; Ornstein-Uhlenbeck process; Dambis, Dubins, and Schwarz Brownian motion; Bessel process; Local asymptotic normality