# Shephard's Lemma and Non-Smooth Integrability Theory

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April 21, 2019

## Abstract

This paper studies a partial differential equation that is called Shephard's lemma in economics. It is known that if the demand function is continuously differentiable, then the local existence of this equation is equivalent to the symmetry of the Slutsky matrix. We extend this result to the class of locally Lipschitz function. Furthermore, we show by using this result that in locally Lipschitz environments, the symmetry and negative semi-definiteness of the Slutsky matrix is a necessary and sufficient condition for ensuring the ability of reverse calculation to a utility function.

**Keywords**: demand function, integrability, income-Lipschitzian, expenditure function, Shephard's lemma.

**JEL Classification Numbers**: D11, C61, C65.

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