A New Folk Theorem in OLG Games

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Overlapping generations (OLG) games have been widely studied to investigate economic organizations whose members change over time. In OLG games, players in the same generation interact for a sufficiently long time, and then are replaced sequentially by successors in the next generation. Kandori (1992) first proved the folk theorem in general n-person OLG games, which stated that the pair of players in each generation can obtain any payoffs in V^*, the convex combination of feasible one-shot payoffs with individual rationality. Following him, Smith (1992) proved several versions of folk theorems with stronger results.

In the analyses of Kandori (1992), Smith (1992), and subsequent studies on OLG games, the characterization of equilibrium set of payoffs is restrictive. They showed that players can obtain any payoffs in V^* without discounting, and that the result is robust against low discounting. However, this does not mean that players cannot attain payoffs outside V^*: when they discount the future, intertemporal trades of payoffs using the difference of time preferences can be possible.

Based on this issue, we recently have shown in Morooka (2020) that in 2-person OLG games with discounting, players can obtain payoffs outside V^*. In the present paper, we extend the result of Morooka (2020) to general n-person OLG games. Our result shows that under the full dimensionality of V^*, any payoffs in the cubic hull of V^*, that is, any payoffs in the smallest n-dimensional cube that contains V^*, are obtainable by the pair of players in each generation.

The main difference between Morooka (2020) and this work is in rewarding players after punishments. In 2-person games, mutual minimaxing is available for punishment against deviations by one player. When there are 3 or more players in one-shot games, they may want to quit punishment because their payoffs during punishment are less than their minimax payoffs. Players must be rewarded with appropriate payoffs after the punishment as an incentive, which requires the full dimensionality of feasible one-shot payoffs with individual rationality.

In order to obtain this result, players' strict discount is essential. In our study, the rate of discounting has two different roles, which can be seen as a trade-off. First, players must be so patient that do not deviate, the argument of which is seen everywhere in the literature on repeated games. At this point, we do not want players to discount the future. The second role, which is our original one, is that it must diminish the payoffs in players' later days. If players do not discount the future, they fail the intertemporal trades of payoffs.

It must also be noted that there is an order in the choice of parameters; we must fix the discount first, and then choose players' lifespan, depending on the discount. This is because, as the discount rate approaches 1, we need a longer lifespan in order for players' continuation payoffs after they become older to diminish sufficiently. This order is inverse to that of the non-uniform OLG folk theorem in Smith (1992), where he first fixes players' lifespan, allowing players to punish deviations strictly under no discount, and then chooses the discount with which the punishments are still available.

In relation to infinitely repeated games, our logic is compared to that of Lehrer and Pauzner (1999). They studied two-person infinitely repeated games with different discounting between players, and showed that players can obtain equilibrium payoffs outside the set of feasible one-shot payoffs V. In their model, the player who evaluates future payoffs at a higher rate gives payoffs to his opponent with less patience first, and is rewarded later. This allows players to obtain higher payoffs outside V. In Lehrer and Pauzner (1999), however, there is an asymmetricity of payoffs between players. In our model, on the other hand, all players' payoffs can be raised at no cost.

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