

# Sequential $\varepsilon$ -Contamination with Learning\*

Hiroyuki Kato<sup>†</sup>

Department of Management and Economics, Kaetsu University

Kiyohiko G. Nishimura

National Graduate Institute for Policy Studies (GRIPS)

and CARF, University of Tokyo

and

Hiroyuki Ozaki

Faculty of Economics, Keio University

## Abstract

The  $\varepsilon$ -contamination has been studied extensively as a convenient and operational specification of Knightian uncertainty. However, it is formulated in a static, one-shot economic environment. This paper extends this concept into a dynamic and sequential framework, allowing learning and guaranteeing time consistency of intertemporal decision. We develop the theory of the *rectangular*  $\varepsilon$ -contamination, which can be represented by a sequence of  $\varepsilon$ 's that “contaminates” the conditional principal probability measure. We then compare this sequential (thus closed-loop) rectangular  $\varepsilon$ -contamination with the initial-period one-shot (thus open-loop)  $\varepsilon$ -contamination, which is a straightforward extension of the static  $\varepsilon$ -contamination.

**JEL codes:** C61, D81, D83

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<sup>†</sup>Corresponding Author.